PACE-2021
International Congress on the Phenomenological Aspects of Civil Engineering
Research Article 20-23 June 2021

Reliability Analysis of Bridge with Monte-Carlo Simulation

Mehmet F. Yilmaz*[1]
[1]Department of Civil Engineering, Ondokuz Mayıs University, Kurupelit 55220, Samsun, Turkey
Corresponding Author E-mail: mehmetfatih.yilmaz@omu.edu.tr Corresponding Author ORCID: 0000-0002-2746-7589

Keywords

Reliability Analysis,
Bridge Safety,
Monte-Carlo Simulation.

Abstract

Transportation systems and bridges must continue to be used safely to maintain society's development and economic growth. For this reason, the structural capacity of these bridges should be determined more realistically and accurately. Reliability analysis determines the probability of failure of a bridge in a statistical way. Existing specifications used a semi-probabilistic approach to calculate the reliability of the bridge and determined a threshold to sustain the safety of the bridge. The development in civil engineering and the more information about the transportation system's material properties and loading conditions make the statistical models more accurate. The advance in technology also makes it possible to apply probabilistic reliability analysis of the bridge. These studies use Monte-Carlo simulation and generate a probabilistic reliability analysis of selected RC box girder bridges. Material properties of the bridge were determined by the experimental data, and normal and log-normal distributions were used to represent the concrete and reinforcement. Normal distribution was used to simulated both dead and live load on the bridge. Bridge collapse loads were determined by the limit analysis theory and simulated. Finally, reliability of the bridge was determined. The results and methodology were discussed in detail.

1.Introduction

An important part of the infrastructures take place on transportation systems have been serving for more than half a century. In developed countries such as the USA, a significant part of these bridges and infrastructure system have either completed their lifetime or are about to complete [1]. The demolition of existing bridges and the construction of new ones creates economically big burdens. However, continuing services of the transport systems without interruption is of great importance considering the economic and social impacts. The traffic loads are increase over time and the structural material properties and strength of the bridges are changed. For this reason a probabilistic approach that represented the changing loading condition and structural properties should be used the determine the reliability of the bridge.

Bridges are exposed to many different deterioration processes during serves life such as corrosion, spills in concrete section, fatigue and cracks[2]. Deterministic expression of these deterioration process is not possible for many cases but some empirical formulas were obtained with the help of the probabilistic methods[3–5]. Furthermore, there are significant effort to determine the deterioration processes of the bridge during serves time and significant data obtained by the visual inspection and structural health monitoring (SHM) activities. Both these information make it possible to generate more realistic simulation of the bridge with the MSC and statistical distribution[6–9]. The obtained information is very important to reliability analysis of existing bridge. Reliability of bridge are determined by the semi-probabilistic approach presented in the existing specifications [10]. The advance in the computer technology and developments in the scientific studies allow engineer to determine the reliability index with a full-probabilistic approach for both new and existing bridges [11–14].

In these study, the advances of Monte-Carlo simulation are used to determine the reliability index of a selected RC box girder bridge with a full-probabilistic approach. Load bearing capacity of the bridge are determined with the limit analysis and moment curvature relationship is generated with OpenSees software. By the way plastic moment capacities of the deck section are determined. Load multiplier are simulated with the full-probabilistic approach and reliability of the bridge are determined.

2. Ultimate Capacity of Bridge

2.1. Limit Analysis of the bridge

The energy consumed by the plastic deformation of the section, and external load must be equal to each other. Rigid perfectly-plastic constitutive laws form the basics of limit analysis. Yielding criterion, starting point of plastic flow, and flow rule, plastic strains are increased correlated to the stress state. Using this theorem limit analysis determines load carrying capacity of structure with a load multiplier. There was two basic approach to calculate the limit analyses; kinematic approach (upper bound) and static approach (Lower bound)[15]. The kinematic approach assumes that the considering materials are perfect plastic and geometry changes are insignificant at the limit load. [16]. In this method, all the possible collapse mechanism is examined, and the minimum load multiplier gives the critical limit load. Comparing bending moment diagram for the collapse load and check, whether in any section plastic moment capacity have been exceeded. If not, the selected mechanism gives the critical collapse mechanism. The static approaches (lower bound) bending moment diagrams are generalized and the loads are increased until first plastic hinge formation will be obtained. Then with changing the structural model with the new plastic hinge formation and bending moment diagram of the new model analyses again with increases the load to examine the second plastic hinge formation and repeat it until the system became unstable. At this point, critical load multiplier is determined. Plastic hinges upon formation as the collapse mechanism of selected two span continuous bridge was shown in Figure 1. α_1 and α_2 shows the rotation at the end support and mid support, v shows the vertical displacement at the point of plastic formation, x shows the position of the plastic hinges and L shows the length of the bridge span.

The virtual work equation could be derived for distributed load as shown in the Eq 1-3. The works done by inertial force are expressed as multiplication of moment capacity of section and matching rotation. The rotation of plastic hinges is derived considering the plastic hinges position x as shown in Eq1. The works done by exterior load are expressed as multiplication of exterior load derivation matching displacement that is calculated according to (x) as shown in Eq2. ζ represent the relation between x and L. ϕ shows the load multiplier that can be calculated by Eq3. Position of plastic hinges can be determined by minimizing the ϕ load multiplier for x plastic hinges position change over 0 (end support) to L (mid support).

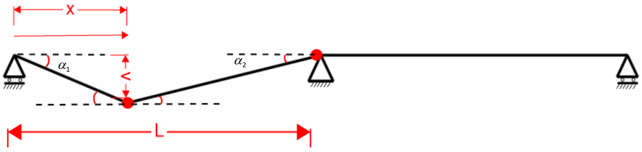


Figure 1 Limit Analysis Collapse Mechanism of Bridge

$$\alpha_1 = \frac{v}{x} \qquad \alpha_2 = \frac{v}{L-x} \tag{1}$$

Work done by external load

$$\int_0^x (G+\phi Q_1)\cdot\alpha_1\cdot x dx + \int_0^{L-x} (G+\phi Q_1)\cdot\frac{v}{(L-x)}\cdot\alpha_2\cdot(L-x)dx \tag{2}$$

$$(G+\phi Q_1)\frac{1}{2}\cdot v\cdot\alpha_1\cdot L$$

Work done by plastic deformation

$$M_p^{+}\left(\frac{v}{x}+\frac{v}{L-x}\right)+M_p^{-}\left(\frac{v}{L-x}\right)\rightarrow\left[M_p^{+}\left(\frac{L-x+x}{x(L-x)}\right)+M_p^{-}\left(\frac{1}{L-x}\right)\right]\cdot v \tag{3}$$

$$\zeta=\frac{x}{L}\rightarrow\left[\frac{M_p^{+}+M_p^{-}\cdot\zeta}{\zeta\cdot(1-\zeta)}\right]\cdot\zeta\cdot\alpha_1$$

Calculating the load multiplier

$$\phi=\frac{\left[\frac{M_p^{+}+M_p^{-}\cdot\zeta}{\zeta(1-\zeta)}\right]\cdot\zeta-G\cdot\frac{L^2}{1}\cdot\zeta}{Q_1\cdot\frac{L^2}{2}\cdot\zeta} \tag{4}$$

2.2. Description of Bridge

A two-span continuous box girder bridge with 30 m length is selected as case study. At the two ends of the bridge, sliding bearings are positioned and at the middle of the bridge fixed bearing is positioned. With the help of two sliding, bearing no inertial force is occurred because of temperature change and elongation and shortening of a bridge. Cross-section of the bridge is composed of a RC box girder with 12m with deck and 2 m height. There is two different reinforcement positioned on the bridge as a result of changing positive moment direction at the middle support. For the 25 m parts from the bridge end support. The more reinforcement positioned at bottom is used to carry positive moment. For the 10-meter section on the bridges middle bearing the more reinforcement positioned at the top of the section.

2.3. Bridge Material Properties

Material properties of Bridge's deck determined by experimental results. Yielding strength of reinforcement is $F_y = 500Mpa$ with coefficient of variation $\delta = 0.05$. The inertial moment of the cracked RC section is defined as $I_{cr} = \alpha I$ with α random variable with Beta distribution $B(a=4, b=2)$ bounded with $\alpha_{\min} = 0.3$ and $\alpha_{\max} = 0.7$.

Mean and standard deviation of the Elastic modules and compression strength of the bridge calculated as $\mu_{E_c} = 30092Mpa$, $\sigma_{E_c} = 4775Mpa$, $\mu_{F_c} = 46.09Mpa$ and $\sigma_{F_c} = 5.048Mpa$. χ^2 and K-S test are applying to determine goodness of fit of the calculated distribution. Both tests give a valuable result for normal and log-normal distribution. The concrete material properties are assumed to be normally distributed in this study. Moreover, regression analysis is conducted to show the relation between elastic modules and compression strength as shown in Figure 2. Dead load and live load are assumed to be distributed on the bridge and the mean values and coefficient of variation of the Dead and Live load considered as $\mu_G = 227.2kN/m$, $\delta_G = 0.05$, $\mu_Q = 140.8kN/m$ and $\delta_Q = 0.2$.

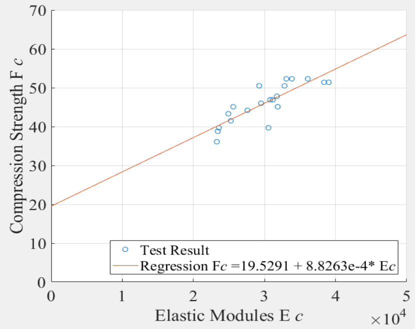
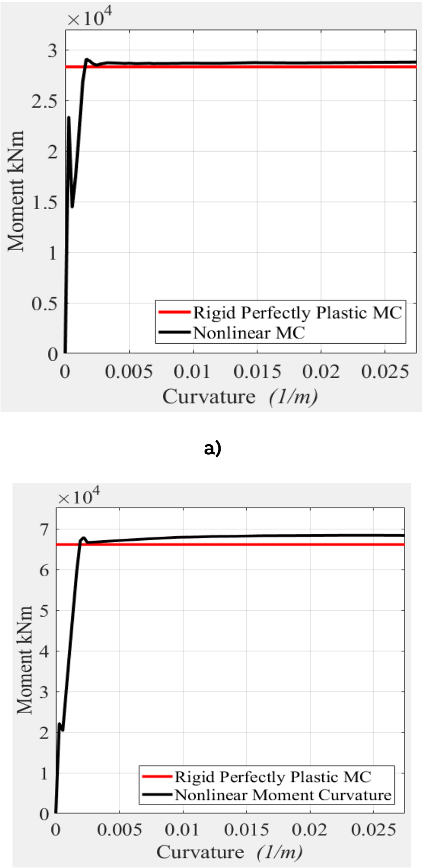


Figure 2 Regression analysis of Elastic modules and ultimate strength of Concreate Bridge Material.

2.4. Moment curvature analysis of box girder bridge

Moment curvature relations of the box girder are modelled using OpenSees finite element software. μ target ductility of sections is assuming as 15 and the initial yielding rotations of the section are calculated with an approximated approach. Concreate02 and steel01 material are used to define material properties of section. Box girder bridges are modelled with help of fiber section and layer command. Moment curvatures are obtaining by increasing the rotation up to the target rotation and nonlinear moment curvature relation obtained. However, limit analysis assumes the sections as rigid perfectly-plastic. A linearization of the moment curvature is necessary to apply the limit analysis on the bridge. In this study, an approximate method is used to determine the linear moment curvature relationship. The area under the linear moment curvature and the nonlinear moment curvature are equalized. Thus, the energies consumed by the two curves are equalized. Both linear and nonlinear moment curvatures are represented in Figure 3. The moment curvatures are derived as an example for mean (f_c)concreate compression and (f_y)steel yielding strength.



a)

b)

Figure 3 Nonlinear Moment curvature relation and Rigid Perfectly Plastic MC a) Section at the span, b) Section at the support

3. Monte Carlo Simulation and Reliability Analysis

Simulation proses duplicates behavior of existing or designing system. It allows the engineer to experience the behavior of the system either better understand the system or further management. The main advantages of simulation are a way to understand the essential component of the system how they response and how they behave in the future. Properties of simulation outputs are determined by input and transfer function. When the transfer function is simple, the properties of output can be calculated analytically. However, in many cases because of different uncertainties and complex actions, it is impossible. In this case simulation, technique gives a better solution with a reliable result depends on the number of simulations. When the number of simulations goes to infinite, the results goes to exact, which is impossible to compute. Therefore, an adequate number of simulations should be determined in using simulation.

Based on structural reliability theory in which R and S are capacity and load and probability of failure are determined by $P_f[R/S]=(R-S)<0$. Reliability index can be expressed as $\beta=\Phi^{-1}(P_f)$. According to the normal distribution assumption, the reliability Index can be determined with the following expression [17].

$$\beta=\frac{\mu_R-\mu_S}{\sqrt{{\sigma_R}^2+{\sigma_S}^2}} \qquad 5)$$

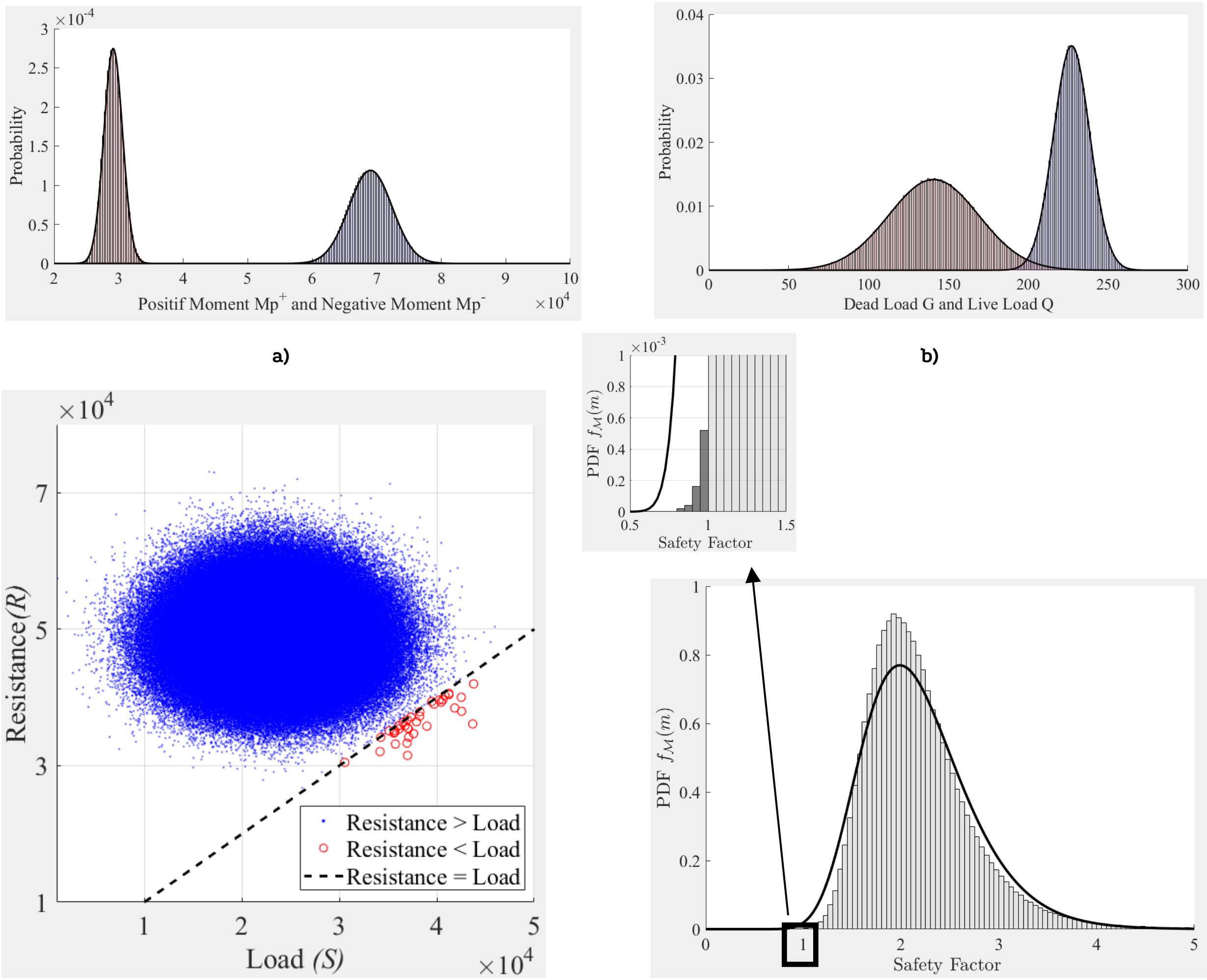
Where μ_R and μ_S are mean values of resistance and load respectively, and σ_R and σ_S are standard deviation of resistance and load respectively.

In Monte Carlo simulation, basic random variables $X=[X_1,X_2,....,X_n]^T$ generated in accordance with their $i=1,2,...,n$ marginal density function $f_{X_i}(x_i)$ and repeated analysis are carried to determine the outcomes [18]. Simulating Resisting R and Load S distribution of the both parameters can be obtained with taking difference of the two parameters for the all simulation sample and number of failures can be obtained. Using the number of failure, total number of simulation probability of failure is calculated as shown in Eq6.

$$P_F=\Phi(-\beta)\cong\frac{N_{fail}}{N_{simulation}}$$
$$\beta\cong-\Phi^{-1}\left(\frac{N_{fail}}{N_{simulation}}\right)=\Phi^{-1}\left(1-\frac{N_{fail}}{N_{simulation}}\right) \qquad 6)$$

If the number of the simulation goes to infinite, the β goes to exact solution.

For bridge structure, the expected reliability index is about 3-3.5 for many cases. But the reliability index of the structure or structural component, mostly larger than these values. Probability of failure for $\beta=3$ calculated as $P_F=\Phi(-\beta=3)=4.4\bullet10^{-3}$ so 10^4 simulation seems to be likely enough to calculate the probability of failure but for $\beta=6$ probability of failure, calculated as $P_F=\Phi(-\beta=6)=6.07\bullet10^{-9}$ so to simulate these at least 10^{10} simulation is require which is too much to calculate and time consuming. For these reasons to calculate, the higher reliability index approximation method can be used. Defining marginal distribution as $M=(R-S)$ and reliability index can be expressed as $\beta=\mu_M/\sigma_M$, The mean values and standard deviation of the marginal distribution can be calculated at the step of simulation and by the way β can be calculated for each simulation. Figure 6 shows an example of these types of calculation. With the increasing of the number of simulations, the reliability index approximates a constant number that is much smaller than the simulation required for determine failure. Mostly normal or log-normal distributions are used to represent the marginal distribution. However, in some complicated case both distributions cannot enough to represent the marginal correctly, and an expected error is occurred.



a)

b)

c)

d)

Figure 4 a) Bridge Span Moment Capacity Distribution, b) Bridge Dead (G) and Live (Q) load distribution, c load factor scatter plot, d) load factor distribution.

Another way to decrease the number of simulation is reduction variance technique [18]. Important sampling (IS) technique is one of these. The technique uses unequally weighted observations and with the small number of observation the probability of failure can be calculated [19–21].

The parameter G , Q_1 , concreate Elastic modules E_c and concreate compression strength F_c are normally distributed. Reinforcement steel yielding strength F_y are log-normally distributed. In order to obtain the M_p moment capacity of the box girder, Monte Carlo simulations are used. Moreover, load multipliers are simulated based on MCS with simulating both dead and live load and M_p moment capacity.

4. Reliability Analysis of Bridge

4.1. Simulating Load Multiplier

Total length of bridge spans is 30m and for first 25m from the end support to the mid support the more reinforcements are applied on the bottom. The 10-meter section on the middle support of the bridge the more reinforcement is formed at the top of the section. Both moment capacities of two reinforcement details deterministically calculated with simulating the concrete compression strength F_c and reinforcement yielding strength F_y. Mean values and standard deviation of both section's moment capacities are calculated as $\mu_{Mp_{s1}} = 29170Mpa$, $\sigma_{Mp_{s1}} = 1451Mpa$, $\sigma_{Mp_{s2}}$ $3352Mpa$, $\mu_{Mp_{s2}} = 69048Mpa$ and the histogram of moment capacity distribution of the sections are shown in Figure 4a. Increasing reinforcement in the tension increases the moment capacities and standard deviation of the section.

Both dead load and live load are simulated using MCS random number and generated simulation are presented in Figure 4b.

ϕ load multiplier is calculated using limit analysis expression and simulation. By leaving alone ϕ , the load multiplier can be easily determined (See Eq3). ϕ <1 is examined in simulation to determine the failure probability of the bridge. The simulation results are shown in Figure 4 c and d. one million sample simulation is generated at the beginning to investigate the accuracies of the model and 37 failure are visualized so the probabilities of failure of the bridge are determined as 3.7×10^{-5} .

4.2. Determining Reliability Index for Bridge

Load multiplier distribution and generated log normal distribution models are shown in Figure 5. The log-normal pdf function has good accuracy of the safety factor distribution with an small error. So, the probabilistic models include some expected error.

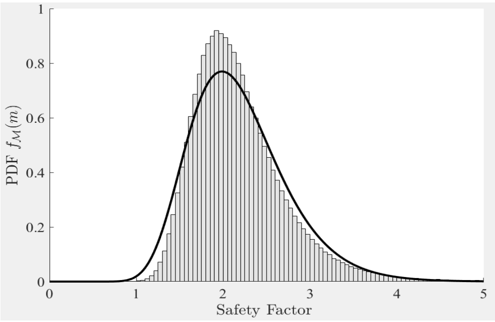


Figure 5 Safety Factor distribution

The β reliability indexes are determined with log-normal distribution approximated approach, classical MCS and importance sampling technique. As mentioned before the log-normal distribution does not satisfy the load multiplier distribution very well so there is some valuable difference between the log-normal modal safety factor and MCs safety factor. Therefore, using the MCS gives us more accurate results as shown in the Figure 6. However, it needs large number of samples. For this example, 1 million sample is enough to calculate the reliability index with MCS but for the higher reliability index level, the number of simulation needs to be increased. Moreover, computational analysis does not cost-effective for such a huge sample of simulation for further analysis such as Life-cycle assessment of the bridge. In these cases, importance sampling technique could be used to reduce the number of simulation and get an accurate result. The blue lines shows the result obtained with importance sampling technique and the results are approximate with the 10^5 sample which is more useful. On the other hand, variance of the IS method is much more than MCS, therefore the results obtained by fewer simulation samples should be verified with sensitivity analysis.

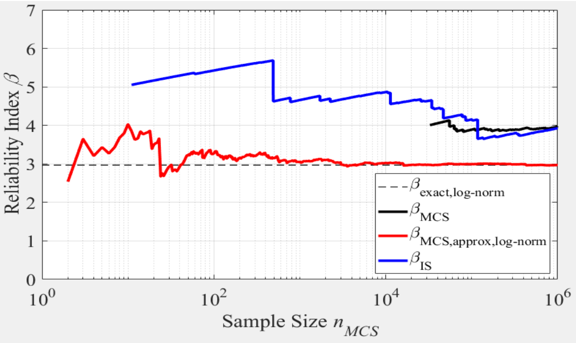


Figure 6 Sensitivity analysis of Beta Reliability index over MCS

Conclusion

In these study, A full-probabilistic approach is applied to determine the reliability index of selected RC box girder bridges and following conclusions are obtained.

- Limit analysis was used to determine the bridge collapse load. In the limit analysis, the live load was increased until the bridge collapsed and the load multiplier was determined. Furthermore, the location of plastic deformation calculated with minimizing the load multiplier over the bridge. If this load multiplier is greater than 1 indicates that the bridge carries the load safely. Load multiplier is simulated with the help of MCS and safety index is calculated.
- When the log normal approximation model is used in calculating the safety index, $\beta \cong 3$ is determined for the current bridge. However, as seen in Figure 6, the Log-normal distribution does not exactly match the load multiplier histogram. Therefore, clearly there is some error in calculation of safety index with the log-normal approximated methods. Using MSC method reliability index of bridge was calculated as $\beta \cong 3.92$.
- Sensitivity analysis shows that minimum one million sampling is required to determine the safety index with the MCS method. The number of simulation consume much time for many case therefore, it is aimed to determine the safety index by using fewer samples and variance reduction technique such as important sampling (IS) is used to decrease the sample number. As a results IS technique reduce the number of simulation to hundred thousandth .

Declaration of Conflict of Interests

The author declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1.] ASCE. Report Card for American's infrastructure. Reston. VA.: 2013.
[2.] Ellingwood BR. Risk-informed condition assessment of civil infrastructure : state of practice and research issues. Struct Infrastruct Eng 2005;1:7–18.
[3.] Biondini F, Frangopol DM. Life-Cycle Performance of Deteriorating Structural Systems under Uncertainty : Review. J Struct Eng 2016;142:1–17.
[4.] Biondini F, Bontempi F, Frangopol DM, Malerba PG. Probabilistic Service Life Assessment and Maintenance Planning of Concrete Structures. J Struct Eng 2006;132:810–25.
[5.] Apostolopoulos CA, Papadakis VG. Consequences of steel corrosion on the ductility properties of reinforcement bar. Constr Build Mater 2008;22:2316–24.
[6.] Okasha NM, Frangopol DM. Integration of structural health monitoring in a system performance based life-cycle bridge management framework. Struct Infrastruct Eng 2012;8:999–1016.
[7.] Zhang B, Benmokrane B, Nicole J, Masmoudi R. Evaluation of fibre optic sensors for structural condition monitoring. Mater Struct Constr 2002;35:357–64.
[8.] Casas JR, Cruz PJS. Fiber optic sensors for bridge monitoring. J Bridg Eng 2003;8:362–73. doi:10.1061/(ASCE)1084-0702(2003)8:6(362).
[9.] Inaudi D, Glisic B. Overview of fibre optic sensing applications to structural health monitoring. Thirteen. FIG Symp. Deform. Meas. Anal. Lisbon, LISBON May 12-15: 2008.
[10.] Akgül F, Frangopol DM. Bridge rating and reliability correlation: Comprehensive study for different bridge types. J Struct Eng 2004;130:1063–74.
[11.] Sommer AM, Nowak AS, Thoft-Christensen P. Probability-Based Bridge Inspection Strategy.pdf. J Struct Eng 1993;119:3520–36.
[12.] Kuroda R, Nishio M. Reliability assessment of an existing steel plate girder bridge using posterior distributions of model parameters. J Japan Soc Civ Eng 2020;8:241–54.
[13.] Nguyen T. Reliability Assessment of Steel-Concrete Composite 2020;2020.
[14.] Tabsh BSW, Nowak AS. Reliability of Highway Girder Bridges. J Struct Eng 1992;117:2372–88.
[15.] Biondini F. Probabilistic Limit Analysis of Framed Structures. 8 th ASCE Spec. Conf. Probabilistic Mech. Struct. Reliab., 2000, p.

273–9.
[16.] He S, Ouyang C, Luo Y. Seismic stability analysis of soil nail reinforced slope using kinematic approach of limit analysis. Environ Earth Sci 2012;66:319–26.
[17.] Caglayan O, Ozakgul K, Tezer O, Uzgider E. Evaluation of a steel railway bridge for dynamic and seismic loads. J Constr Steel Res 2011;67:1198–211.
[18.] Biondini F. Use of Simulation in Structural Reliability. Struct. Congr. 2008, 2008.
[19.] Melchers RE. Importance Sampling in Structural Systems. Struct Saf 1989;6:3–10.
[20.] Infanger G. Monte Carlo (Importance) Sampling Within a Benders Decomposition Algorithm for Stochastic Linear Programs. Ann Oper Res 1992;39:69–95.
[21.] Oh M, Berger JO. Adaptive importance sampling in monte carlo integration. J Statictical Comput Simul 1992;41:143–68.