Reduction of Wave Forces on a Rigid Breakwater by a Perforated Thin Barrier over Step Bottom

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Abstract

The study deals with the interaction of water waves with the perforated thin barrier placed in front of a rigid wall in the presence of step bottom. To include the effect of wave heights on the wave energy dissipation, a nonlinear pressure drop condition on the perforated barrier is considered. Further, the associated boundary value problem is solved numerically using the iterative boundary element method. Finally, the reflection coefficient and wave forces are evaluated and analyzed to study the various characteristics of the trapping problem. The study concludes that the wave forces on the rigid breakwater reduce significantly with an increase in the porosity of the perforated thin barrier.

1. Introduction

Perforated thin barriers are the most commonly used coastal structures to mitigate the wave-induced forces and incident wave energy dissipation. These perforated barriers are highly preferred due to their simple design, construction conveniences, and cost-effectiveness [1].

There are many experimental and theoretical investigations carried out by several researchers related to wave interaction with thin and thick porous barriers. Among them, some notable works are Porter and Evans (1995) [2] studied the scattering of water waves by thin vertical walls of different structural configurations using Galerkin approximations in infinite-water depth. Li et al. (2006) [3] studied the effect of the relative thickness of the porous plate on the resistance coefficient. It was observed that when the relative thickness of the plate is reduced, the resistance coefficient significantly increased. Liu et al. (2012) [4] used the eigenfunction expansion to study oblique wave interaction with an infinite array of perforated screens placed in front of the rigid caisson to mitigate the horizontal forces on the caisson. The study highlighted that the number of the reflected wave and propagation waves is not dependent on the number of porous barriers. An and Faltinsen (2012) [5] presented wave scattering by a submerged perforated plate in finite and infinite water depths using the time-efficient iterative semi-analytic solution. A perforated plate can effectively reduce the wave loads and reflections than the rigid plate. However, the wave transmissions are more in the case of long waves [5].

Several studies are available on wave interactions with the thin perforated barrier with linear boundary conditions on the plate. In general, the flow through the perforated barrier is represented by a nonlinear boundary condition. Using the nonlinear pressure drop condition on the perforated barrier, [6] studied the interaction between the water waves and perforated screens theoretically and experimentally. It was seen that a perforated barrier with 20% geometrical porosity gives the optimal wave reflection. Liu and Li (2017) [7] used an iterative boundary element method to handle the nonlinear condition on the perforated plate when wave interaction with the partially perforated caisson on the rigid foundation. This nonlinear pressure drop condition signifies the effect of wave height on the wave energy dissipation, which was neglected in the liner version of the pressure drop condition. Recently, Panduranga et al. (2021) [8] studied the gravity wave interaction with multiple surface piercing slotted screens in front of the rigid caisson placed on a porous foundation in the presence of seabed undulations. The physical problem was solved using the iterative boundary element method. A combination of four slotted barriers produces 98% of wave energy dissipation.

In the present study, water wave interaction with a bottom-founded perforated barrier over the step bottom is studied using linear wave theory. A nonlinear pressure drop condition is considered to handle the effect of wave height on wave energy dissipation. The associated boundary value problem is solved using the iterative boundary element method. Various wave and structural parameters are considered to investigate the effectiveness of the perforated barrier, and details are provided in the subsequent sections.

2. Mathematical formulation

In this section, water wave interaction with a bottom standing perforated thin barrier place at a finite distance away from the rigid breakwater is studied under linear wave wave theory. The problem is studied in a 2D cartesian coordinate system in which the z-axis is taken vertically upwards, and the x-axis is taken along the mean free surface. In Fig. 1, $d$ is the length of the thin barrier, $h_1$ is the water depth which changes to $h_2$ at $x = 0$. Further, the distance between the rigid wall and the thin barrier is $w$.
3. Numerical solution using iterative boundary element method (BEM)

In this section, the aforementioned BVP is solved using BEM. Firstly, a set of integral equations are formulated using Green’s second identity for each of the boundaries of the regions \( R_j, j = 1, 2 \). Then, the integral equations are handled for the numerical solution using BEM. Due to nonlinearity in Eq. (4), the conventional BEM cannot be applied to the present problem. Therefore, an iterative BEM is adopted to handle the BC on the perforated thin barrier.

Applying Green’s second identity to \( \phi \) and \( G \) (fundamental solution) over each boundary of the regions \( R_j, j = 1, 2 \), we obtain the following integral equation

\[
\frac{1}{2i} \phi = \int_{\Gamma} \left( \frac{\partial G}{\partial n} - G \frac{\partial \phi}{\partial n} \right) d\Gamma 
\]

where \( n \) represents the unit normal vector (outward). The fundamental solution \( G \) is expressed as

\[
G = \frac{1}{2i} \ln \sqrt{(x-x_0)^2 + (z-z_0)^2 + 2(z-z_0)Y},
\]

and its normal derivative

\[
\frac{\partial G}{\partial n} = n_x \frac{\partial G}{\partial x} + n_z \frac{\partial G}{\partial z},
\]

where \( n_x \) and \( n_z \) are unit normal vector components along x and z directions. Substituting BCs (2)-(7) into Eq. (8), we get the following integral equations (9).

**Region 1:**

\[
-\frac{1}{2i} \phi_1 + \int_{\Gamma_1} \left( \frac{\partial G_1}{\partial n} - ik_0 G_1 \right) \phi_1 d\Gamma_1 + \int_{\Gamma_1} \frac{\partial G_1}{\partial n} \phi_1 d\Gamma_1' + \int_{\Gamma_2} \left( \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \phi_1}{\partial n} \right) d\Gamma_2 + \int_{\Gamma_2} \frac{\partial G_1}{\partial n} \phi_1 d\Gamma_2'
\]

\[
= \frac{1}{2i} \int_{\Gamma_1} \left( \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \phi_1}{\partial n} \right) d\Gamma_1
\]

**Region 2:**

\[
-\frac{1}{2i} \phi_2 + \int_{\Gamma_1} \left( \frac{\partial G_2}{\partial n} + \phi_2 \frac{\partial \phi_1}{\partial n} \right) d\Gamma + \int_{\Gamma_1} \frac{\partial G_2}{\partial n} \phi_2 d\Gamma'
\]

\[
= \frac{1}{2i} \int_{\Gamma_1} \left( \frac{\partial G_2}{\partial n} + \phi_2 \frac{\partial \phi_1}{\partial n} \right) d\Gamma + \int_{\Gamma_1} \frac{\partial G_2}{\partial n} \phi_2 d\Gamma'
\]

In Eq. (10), \( Y = \frac{ni}{2mu} \left( \frac{1}{2} + \frac{1}{4a} \right) + 2C \). The above integral equations are converted into a matrix equation by discretizing the entire boundaries of the regions into a finite number of boundary elements, and the values of \( \phi \) and \( \phi \partial / \partial n \) are assumed to be constant over each element [9]. After obtaining \( \phi \) and \( \phi \partial / \partial n \), the horizontal wave force acting on the thin perforated plate and rigid wall are evaluated using the following formulat.

**Reflection coefficient** \( K_R [9] \) is given by

\[
K_R = -1 + \frac{1}{2i} \frac{2\omega^2}{N_0^2} G e^{ik_0 h} \int_{-h}^{h} \phi^{(-1)}(z) Z_2(z) dx 
\]

where

\[
N_0^2 = \int_{-h}^{h} Z_2(z) dz,
\]

\[
Z_2(z) = \frac{\cosh k_0(z + h)}{\cosh k_0 h}.
\]
Horizontal wave loads on barrier $F_S^x$ and wall $F_W^x$ [9] are given by

$$F_S^x = \frac{\omega}{gh} \int_{x_0}^{x_1} (\phi_2 - \phi_1) n_x \, dx,$$  \hspace{1cm} (12)

$$F_W^x = \frac{\omega}{gh} \int_{x_0}^{x_1} \phi_2 n_x \, dx.$$  \hspace{1cm} (13)

4. Results and discussions

For the computational purpose the wave and structural parameters are fixed as following: $h_1 = 10m$, $h_2/h_1 = 0.5$, $d/h_2 = 0.5$, $W/h_1 = 1.0$, $L/h_1 = 2.0$, $a = 0.4$, $b = 0.01 h_1$, unless otherwise mentioned.

Figs. 2(a)-2(c) show the variation of $K_R$, $F_S^x$, and $F_W^x$ versus $k_0 h_1$ for different values of the porosities of the perforated barrier. $K_R$ decreases with an increase in the porosity of the perforated barrier. The reason is more amount of incident wave energy being dissipated by the perforated barrier. Further, $k_0 h_1 > 1.5$, i.e., short wave regime, the reflection coefficient increases for higher values of the porosity. In Fig. 2(b), the non-dimensional horizontal wave force on the rigid wall is plotted for various porosities of the perforated barrier. It is seen that the wave forces on the rigid wall decrease significantly with a change in porosities. However, for a certain range of wave frequencies, the non-dimensional wave forces on the rigid wall increase for larger geometrical porosity values. This is mainly the transmission of wave energy through the perforated barrier. Further, the non-dimensional wave forces on the perforated wave barriers decrease for a certain range of wave frequencies. This is expected since a major portion of wave energy is dissipated by the perforated barrier.

Figs. 3(a)-3(c) show the variation of $K_R$, $F_S^x$, and $F_W^x$ versus $W/\lambda$ ($\lambda$ is the incident wavelength) for different values of the porosities of the perforated barrier with $k_0 h_1 = 1.0$.

5. Conclusions

The study presents the reduction of wave loads on the rigid breakwater by the thin perforated barriers. The associated boundary value problem is solved using the iterative boundary element method due to the nonlinear pressure drop condition on the perforated barrier.
It is found that the reflection coefficient decreases with an increase in the porosity of the perforated barrier. This due to dissipation incident wave energy. Further, the non-dimensional horizontal force on both rigid wall and perforated barrier also decrease with an increase in the porosity of the barrier. Due to the continuous interaction of reflected waves by the rigid wall and perforated barrier, the reflection coefficient and wave loads follow an oscillatory manner as the non-dimensional gap between the rigid wall and perforated breakwater. Further, it can also be studied with an increase in the number of perforated barriers for the significant reduction of wave loads on the rigid breakwater.

Declaration of Conflict of Interests

The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References


