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An Efficient Rectangular Flat Shell Finite Element for the Analysis of Thin Shell Structures

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Keywords

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Thin Shell Structures,
Strain Based Approach,
Cylindrical Shell Model,
Experimental Test,
Geometrical Shape.

Abstract

Numerical analysis becomes an efficient practical tool for the analysis of engineering structures whatever the geometrical shape and applied loads. In this paper, a rectangular flat shell element denoted 'ACM_R4SBE5' is presented. This element is constructed by superposition of a rectangular strain-based membrane element 'R4SBE5' and the well-known plate bending element 'ACM'. This element can be used for the analysis of thin shell structures even with a complicated geometrical shape. Validation of the present element is performed by a clamped cylindrical shell model and an experimental test on semi cylindrical shell model. In addition, a comparison of the deflections obtained with other types of shell elements is presented and discussed. According to the results obtained, the formulated flat shell element proved to have a fast rate of convergence and to provide satisfactory results.

1. Introduction

Generally, for design purposes, shell structures are constructed with very complicated geometrical shapes and elements, such as folded plates and edge beams. Additional geometrical problems arise such as when openings, anisotropy or variation of thickness are present. Analytical solutions of practical thin shell structures, particularly those with irregular geometrical shapes, are complex and thus a resort to numerical methods when analyzing them becomes essential. However, for practical purposes the flat element approximation gives generally adequate results and permits easy coupling with edge beams and rib members, a capability usually not present in curved element formulations [1]. In flat shell elements, the coupling between membrane and bending action is accounted for at the integration points due to the varying orientation of the element. For practical analysis of shell structures, such flat plate element assumption is typically acceptable, and has the advantage of ease of modeling with reasonable accuracy. Further, because the membrane and bending stresses within an element are decoupled it is easy to understand and control the behavior of such elements [2].

In this case, the behavior of a continuously curved surface is represented by a surface made up of small flat elements. Intuitively, as the size of the subdivision decreases it would seem that convergence must occur as discussed by Zienkiewics and Taylor (2000) [1]. In this paper, a new flat shell element is proposed and is denoted as ACM_R4SBE5. The element is developed by superposition of the new rectangular membrane element R4SBE5 based on the strain approach and the well-known plate bending element ACM described in detail by Adini and Clough (1961) [3]and Melosh (1963) [4]. ACM_R4SBE5 element is characterized by its simplicity compared to existing elements, without compromising its numerical robustness. Also, the technique of static condensation of a middle node and the new analytical integration employed in the formulation are the new additions that distinguish this element from other flat shell elements presented in previously published works (e.g. Ashwell & Sabir (1972) [5], Sabir & Lock (1972) [6], Belarbi (2000) [7], Batoz & Dhatt (1992) [8].

In the following sections, the formulation of the new element ACM_R4SBE5 is presented, followed by a validation test to show its convergence. The performance of ACM_RSBE5 element is also compared to other quadrilateral shell elements evaluable in the literature. Finally, an experimental work is conducted to confirm experimentally the results and the efficiency of the present flat shell element ACM-R4SBE5.

2. Presentation of finite elements used for the numerical analysis

2.1. Flat shell element "ACM-R4SBE5"

Figs.1 and 2 show the geometry of "R4SBE5" element (Strain Based Rectangular Element with four corner nodes and an internal node), the corresponding nodal displacements and the ACM plate bending element with four nodes and three degrees of freedom per node respectively.

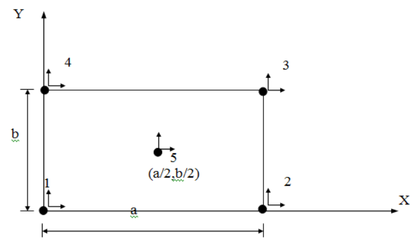

Figure 1. Co-ordinates and nodal points for the rectangular element "R4SBE5"

The displacement fields of the R4SBE5 element are given by the following expressions Hamadi (2006) [9].

$$U = a_1 - a_3 y + a_4 x + a_5 xy - a_7 y^2 (R+1)/2 + a_8 y/2 + a_9 (x^2 - H y^2)/2 \quad (1a)$$

$$V = a_2 + a_3 x - a_5 x^2 (R+1)/2 + a_6 y + a_7 xy + a_8 x/2 + a_{10} (y^2 - H x^2)/2 \quad (1b)$$

with: $H = 2/(1-\nu)$; $R = 2\nu/(1-\nu)$

The displacement fields of the Rectangular plate element 'ACM' element are:

$$W(x,y) = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3 + a_{11} x^3 y + a_{12} xy^3 \quad (2a)$$

$$\theta_x = -a3 + a_5 x + 2 a_6 y + a_8 x^2 + 2a_9 xy + 3 a_{10} y^2 + a_{11} x^3 + 3 a_{12} xy^2 \quad (2b)$$

$$\theta_y = a_2 + 2a_4 x + a_5 y + 3a_7 x^2 + 2a_8 xy + a_9 y^2 + 3 a_{11} x^2 y + a_{12} y^3 \quad (2c)$$

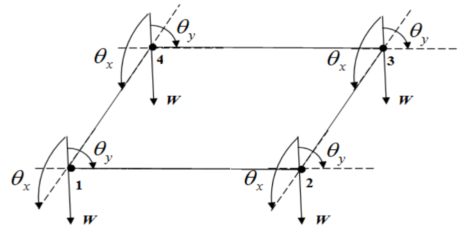

Figure 2. Co-ordinates and nodal points for the rectangular plate element "ACM"

The flat shell element ACM_R4SBE5 is obtained by assembling the two elements R4SBE5 and ACM with addition of an effective rotation θ_z (Fig.3).

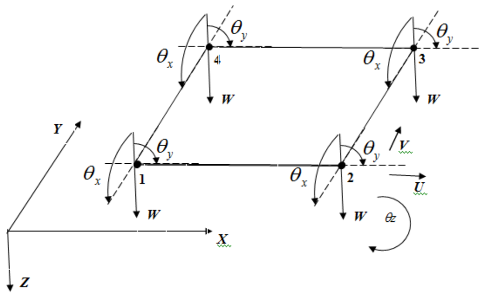

Figure 3. The flat shell element ACM-R4SBE5

The stiffness matrix of the shell element ACM-R4SBE5 is obtained by using the analytical integration of the membrane and bending stiffness matrix. The calculation of the element stiffness matrix is summarized with the following well known expressions:

$$[K_e] = [A^{-1}]^T \left[\iint_S [Q]^T [D] [Q] \, dx \, dy \right] [A^{-1}] \quad (3)$$

$$[K_e] = [A^{-1}]^T [K_0] [A^{-1}] \quad (4a)$$

$$[K_0] = \iint_S [Q]^T [D] [Q] \, dx \, dy \quad (4b)$$

Where: $[D]$ The constitutive matrix,

$[A]$ The transformation matrix,

$[Q]$ The strain matrix,

and $[K_e]$ is the elementary stiffness matrix.

2.2. S4R ABAQUS element

The S4R is a four -node doubly curved element used for thin and thick shells. It has six degrees of freedom at each node (6 DOF/node) [10].

2.3. C3D8IH ABAQUS element

The C3D8IH element is a general purpose linear brick element, with full integration points, hybrid formulation and incompatible modes. The node numbering follows the convention as shown in (Fig.4) [10].

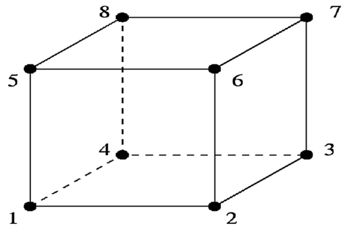

Figure 4. Eight node Brick element

3. Presentation of finite elements used for the numerical analysis

The main purpose of this experimental test is conducted to validate experimentally the presented flat shell element ACM-R4SBE5. Two experimental models are carried out on semi cylinder models; pinned and fixed supports.

- Semi cylindrical shell supported on 4 points "Pinned" CS4P.
- Semi cylindrical shell supported on two ends "Rigid Diaphragms" CSRD

Fig.5 presents the UNIFLEX 300 machine, the shell model setup, the positioning of dial gauges. Figs. 6 (a) and (b) present the different boundary conditions used. The following loads are applied (775 N, 800 N, 825 N, 850 N, 875 N and 900 N) and the results obtained for deflection at point 1 (top centre of the model) are recorded in Table1. It should be mentioned here, that this work is a part of set of experimental tests conducted at Civil Engineering Laboratory at City University of London [11].

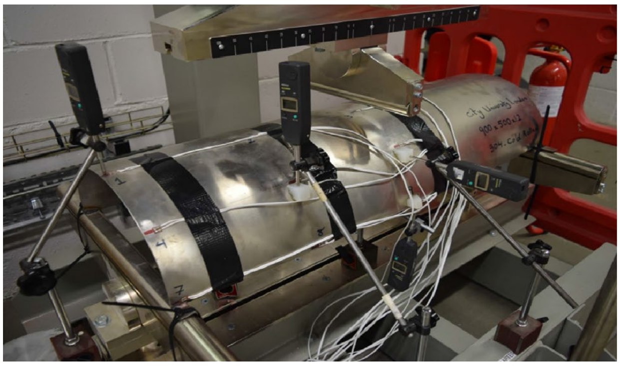

Figure 5. The shell model

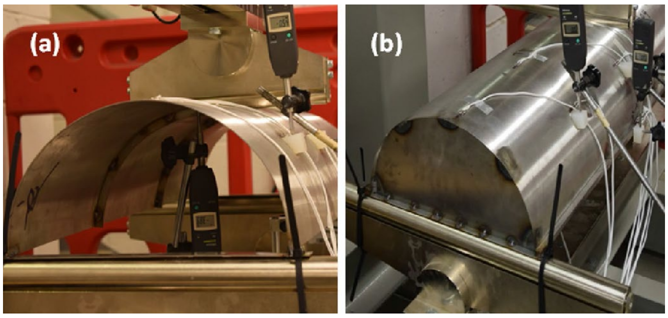

Figure 6. (a) Semi cylindrical shell reposed on four points "Pinned" (b) Semi cylindrical shell reposed on rigid diaphragm "Fixed"

4. Validation of ACM-R4SBE5 element

The performance of the formulated flat shell element ACM-R4SBE5 is evaluated through the well known test problems. In this paper we will present only the well known test clamped cylindrical shell (Figs.7 (a)). This test of thin shells (R/h = 100) is considered by many researchers as a severe test. It makes it possible to examine the aptitude of shell elements to simulate complicated membrane state problems dominated by bending. The dimensions, material properties and loading conditions are shown in Fig.7.

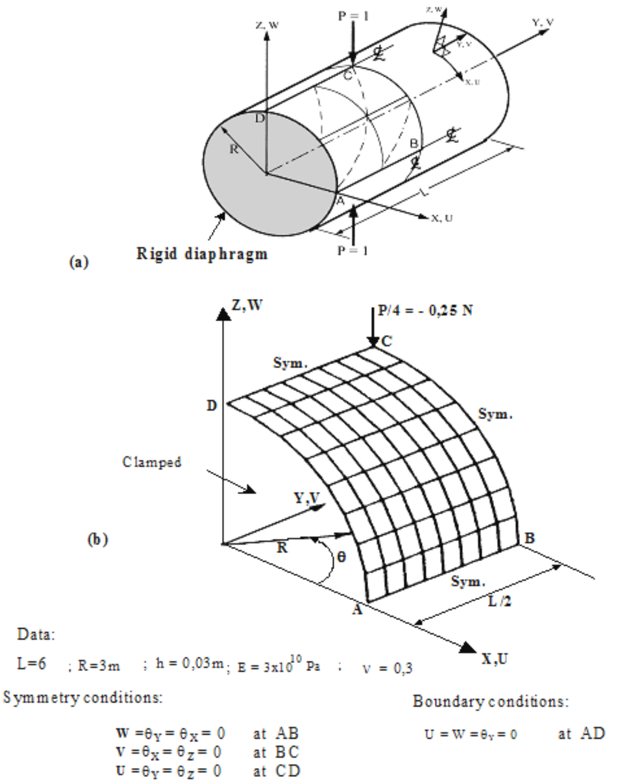

Figure 7. Clamped cylindrical shell

Due to symmetry, only 1/8 of the shell (region ABCD) is considered in the finite-element idealization (Figure 7 (b)). The results obtained for different meshes for both, the proposed ACM_R4SBE5 and the ACM_SBQ4 of Belarbi (2000) [7], are presented in Table 1 and compared to the analytical solution based on the thin shell structures (R/h=100) given by Flugge (1966) [11] below:

$W_C = -W_C\ Eh/P = 164{,}24$ deflection under load P in point C only

Table 1 also summarizes the solution time used in the analysis of the clamped cylindrical shell with different meshes. The processor machine used has the following properties: Intel(R) Core(TM) i3–2330 M CPU@2.2 GHZ RAM: 4.00Go

Table 1. Clamped cylindrical shell, convergence of displacement Wc at point C (normalized values)

Meshes	ACM-R4SBE5	ACM-SBQ4	HEX20	SHB8PS	RESS	Solution time (Sec) ACM-R4SBE5
4x4	0.649	0.618	0.140	0.387	0.112	0.10000
6x6	0.842	0.821	0.328			0.17999
8x8	0.955	0.904	0.523	0.754	0.590	0.26999
20x4	0.984	0.956	0.675			0.28845
16x16				0.940	0.933	
Analytical solution	164.24 (1.00 Normalized results)					

5. Discussion of the numerical results obtained

The results obtained by the present element ACM_R4SBE5 for the vertical displacement at point C (normalized values) as shown in Table 1 are slightly better than those given by the element ACM-SBQ4, and much better than those of the hexahedral element HEX20, the standard full-integration solid element reported in Abed-Meraim, Trinh, and Combescure (2013) [12]. It also performs better than the eight-node solid–shell element SHB8PS (Abed-Meraim & Combescure (2007) [13]. Also, ACM_R4SBE5 performs relatively better than the widely used Reduced Integration Enhanced Strain Solid-Shell (RESS) element developed by Alves de Sousa et al. (2005) [14]. Furthermore, it significantly converges more rapidly than others elements as shown in Table 1.

6. Comments on the numerical and experimental results

Table 2 shows the results obtained from the experimental test, as well as the flat shell element ACM-R4SBE5 and ABAQUS code with meshes of 10 × 10 elements.

Table 2. The deflection diminution percentage using ACM-R4SBE5 element, ABAQUS element and the experimental results for semi cylindrical shells (CS4P and CSRD) at point 1

Load (N)	ACM-R4SBE5		
	Deflection (mm)		Percentage (%)
	CS4P	CSRD	
775	5.061	1.064	78.97649
800	5.224	1.098	78.981623
825	5.387	1.133	78.96789
850	5.551	1.167	78.976761
875	5.714	1.201	78.981449
900	5.877	1.236	78.96886
	S4R ABAQUS		
775	5.08	1.087	80.7165358
800	5.241	1.122	80.70979
825	5.403	1.157	80.69591
850	5.564	1.192	80.69734
875	5.726	1.227	80.684597
900	5.888	1.262	80.216033
	Experimental solution		
775	4.648	1.438	69.06862
800	5.098	1.727	66.12397
825	5.606	1.831	67.338566
850	6.060	1.936	68.05805
875	6.422	2.013	68.654625
900	6.822	2.103	69.173263

The results indicate that the finite element models are slightly under-predicted the displacements at higher applied loads. This can be attributed to the settlement of the test setup observed in the experiment. From Table 2, it is also, observed that the deflection diminution percentage of the experimental results in the presence of the rigid diaphragm is almost 68%; that means the rigid diaphragm minimized the vertical displacement at point 1 by 68%, which is an excellent contribution compared to shell supported on 4 points. Whereas, the deflection diminution percentage is almost 80% according to both the ACM-R4SBE5 and ABAQUS results, this indicating that both models resulted in reasonable simulation of the behavior.

7. Conclusion

The proposed element ACM_R4SBE5 has the advantage of being simple in form and uses only the four nodes and five essential degrees of freedom.

According to the validation tests and the results obtained with other finite elements in the literature, the efficiency of the ACM_R4SBE5 element is confirmed.

The results obtained compared to the other finite elements confirm the fast convergence rate of the element. Furthermore, a correlation study with an experimental semi cylinder shell model confirms also the accuracy of the proposed element.

ACM_R4SBE5 element can be used for the analysis of thin shell structures, even those with complex geometrical shapes.

A high percentage of deflection reduction can be achieved with Rigid Diaphragms. This is due to the fact that when using rigid diaphragms, the effect goes from the skin to the curved boundaries of the cylinder then to the rigid diaphragms. So, the cylinder with rigid diaphragms can support much higher loads with smaller deformations.

The current formulation is carry out for linear analysis, the extension to non linear geometric and material will be a very interesting subject focuses in future work.

Declaration of Conflict of Interests

The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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