**Static and Dynamic Investigation of Structure-Foundation-Reservoir Problem Utilizing Finite Element Method**

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**Abstract**

In this article, linear elastic static and linear elastic dynamic analysis of two-dimensional structure-foundation-reservoir problem is discussed. Mechanical interaction between structure-foundation-reservoir and discretization of these regions are simulated by reliable numerical Finite Element Method. The reservoir region which incorporates the compressibility effect and surface sloshing motion of water is formulated by two-dimensional Lagrangian fluid finite elements. The dynamic responses of the structure under a given ground motions are obtained in three conditions: structure-reservoir interaction with rigid foundation, structure-reservoir interaction with finite region of flexible foundation and structure-reservoir interaction with including the infinite region of flexible foundation. During dynamic analysis, only the radiation conditions toward infinity and energy dissipation in foundation is simulated considering infinite elements in the infinite region of the flexible foundation. The results of dynamic analysis obtained from structure-foundation-reservoir interaction are shown without considering the radiation conditions in the infinite region of the constant depth reservoir. And only the static and dynamic displacement responses of the crest are given in this paper. All steps of static and dynamic analysis are performed using FORTRAN 90 coding language. The static and dynamic responses are validated and compared with the results of other research papers.

**Keywords**

| Lagrangian Fluid Element, Infinite Element, Structure-Foundation-Reservoir Problem, Static And Dynamic Analysis, Radiation Condition. |

**1.Introduction**

In civil and mechanical engineering problems, the dynamic analysis of coupled structure-reservoir-foundation problem is challenging due to the following factors: dynamic interaction between structure-reservoir considering the hydrodynamic pressures, radiation of waves to far end reservoir boundary and compressibility effects of water, dynamic interaction between structure-foundation considering the flexibility, inertia, material damping, radiation damping and infinite region of foundation, dynamic interaction between reservoir-foundation considering the absorption of reflected waves at the reservoir boundary. In the past few decades, some numerical methods like Finite Element Method [1–7]. Boundary Element Method [8] and coupled FEM-BEM Method [9–10] have been evolved to decipher the 2D and 3D dynamic responses of this kind coupled complex problems.

During past years, Chopra and his colleagues have implemented outstanding works and developed some software to analyze the static and dynamic responses of dam-reservoir-foundation problem in time domain and frequency domain utilizing Finite Element Method [1–7].

Generally, the static and dynamic responses of this type coupled problems could be obtained using Lagrangian and Eulerian approaches. In the Lagrangian approach, the nodal variables in structure, foundation and water (fluid) are displacements; while in the Eulerian approach, the nodal variables in structure and foundation are displacements; pressures or velocity potentials are nodal variables in water.

Many researchers have commonly utilized Eulerian approach to analyze the structure-reservoir-foundation interaction problem. Because the variables in water, foundation and structure are different, interface equations required at the structure-water interface; water-foundation interface interaction are also developed by many investigators [11–17].

Lagrangian fluid finite element is easy to code as well as general finite elements due to the same variables in the fluid, structure and foundation region. And it is the advantage of this approach. But, this approach encounters some numerical problems during analysis (e.g. hour-glass mode or zero-energy modes). These zero energy-modes mostly come out from zero shear modulus combined with reduced numerical integration method in addition with finer mesh of the numerical model [21]. In the literature, some alternatives that can eliminate these numerical problems are presented by many researchers [18–21]. Due to the compatibility of variables in structure, reservoir and foundation regions, many researchers have used Lagrangian approach to solve this kind of coupled problems [22–24].

In coupled structure-reservoir interaction problem, structure-foundation interaction problem and reservoir-foundation interaction problem, the radiation of reflected waves to infinite medium is one of the most important factors which can merely impact the dynamic behavior of the coupled system. During dynamic loads, hydrodynamic forces are reflected to infinity. If the reservoir bed and far end reservoir boundary considered to be rigid, then the hydrodynamic forces will be over-estimated. Therefore, this will give inappropriate dynamic behavior of the system, which does not determine the actual behavior of the coupled system. It is also a challenging problem need
to be developed appropriately. In the literature, many investigators have developed some absorptive elements (finite elements) and absorptive boundary (artificial boundary) to soak in the reflected waves at the reservoir bed and at the reservoir far end boundary [25–35]. Additionally, this condition should be considered for unbounded foundation medium.

The objectives of this research paper are firstly, to discuss the dynamic analysis of structure-foundation-reservoir interaction problem using Lagrangian approach; secondly, to use an appropriate model of absorptive boundary at the far end boundary of flexible foundation and constant depth reservoir; thirdly, to develop a powerful and stable FORTRAN 90 code for the static and dynamic analysis of 2D structure-foundation-reservoir interaction problem. All steps of static and dynamic analysis are programmed in FORTRAN 90 coding language utilizing the above mentioned Lagrangian approach for fluid domain and standard Finite Element procedure for structure and foundation domain. The program has been individually applied on water (fluid), structure-water interaction and structure-water-foundation interaction problems; the results were validated and compared with results of various papers.

2. Lagrangian Approach for Fluid and Deriving the Coupled Equation of Motion

In this approach, because of using the same nodal variables (displacements) in structure, water and foundation regions, there is no need for interface equations. Therefore, the compatibility and equilibrium equations along the interface nodes are automatically provided [21]. In this section firstly, the basic equations for linear-elastic, non-rotational, non-viscous fluid elements which make small displacements will be given [37]. Then move on to the finite element formulation of structure-reservoir-foundation problem. Thus, the pressure in a fluid element can be calculated by,

\[ P = \beta \epsilon_v \]

Where, \( P \), \( \beta \) and \( \epsilon_v \) represent pressure, volumetric modulus of elasticity (bulk modulus), and volumetric strain of the fluid element respectively. The volumetric strain in two-dimensional Cartesian coordinates may be written,

\[ \epsilon_v = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \]

Where, \( \frac{\partial}{\partial} \) denotes the partial derivative of \( i \) displacement component with respect to the \( j \) direction. The rotational characteristic of the fluid element can be restricted using penalty methods [21] which accomplish non-rotational fluid elements. By giving a large restriction parameter to the stress-strain equation of fluid element, the rotations of the fluid element will be constrained. A two-dimensional fluid element has only one rotational equation which can be given by,

\[ W_e = \frac{1}{2} \left( \begin{array}{c} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \end{array} \right) \]

Due to restriction of element rotation, the rotational stress can be defined by,

\[ P_k = a W_e \]

In this equation, \( \sigma \), \( e \) and \( C_t \) represent stress vector, strain vector and elasticity matrix (material matrix) of fluid element, respectively. In this section, only the equation of motion for fluid domain is derived step by step using energy methods. Then, the same procedure can be applied again to obtain the equation of motion for solid (structure and foundation) domain. Thus, the total potential energy of the fluid domain consists of strain energy which can be calculated by,

\[ \Pi_e = \frac{1}{2} \int e^T C_t e \, dA \]

And strain energy of the fluid without changing in the fluid volume. This characteristic expresses an important behavior of fluid domain. Generally, this behavior comes out in the form of sloshing waves or up down movements at the free surface of fluid. This type of potential energy of the fluid domain which comes out from surface sloshing motion of fluid, can be calculated by,

\[ \Pi_i = \frac{1}{2} \int u^T g U_d \, dl \]

where, \( \rho_i \), \( g \) and \( U_d \) represent mass density of the fluid, gravitational acceleration and vertical displacements of the fluid free surface, respectively. In equation 6, \( e \) and \( C_t \) respectively define the displacements vector and the elasticity matrix of fluid domain. Therefore, the total potential energy of fluid domain may be written as,

\[ \Pi = \Pi_e + \Pi_i \]

Since the dynamic behavior of the fluid system is considered in this paper, the kinetic energy of the fluid domain should be included which is defined by,

\[ T = \frac{1}{2} \int \rho_i \left( \frac{\partial v^2}{\partial t} + \frac{\partial u^2}{\partial t} \right) \, dA \]

where \( \rho_i \) is mass density of the fluid, and \( \frac{\partial v}{\partial t} \) defines velocity of the point in the \( x \) direction.

By using the fundamental concepts of finite element method, the total strain energy of the fluid domain given in equation 6 can be written as,

\[ \Pi_e = \frac{1}{2} U^T K_e U \]

where \( U \) is the nodal displacement vector and \( K_e \) is the stiffness matrix of the fluid domain. Similarly, the potential energy at the free surface of fluid domain given in equation 7 can be written as,

\[ \Pi_i = \frac{1}{2} U^T S U \]

where \( U \) and \( S \) respectively represent the vertical nodal displacement vector and stiffness matrix of the free surface of the fluid domain. Also, the kinetic energy of the system given in equation 9 may be written as,

\[ T = \frac{1}{2} U^T M_f U \]

where \( U \) and \( M_f \) respectively represent the nodal velocity vector and the mass matrix of the fluid domain.
By substituting the energy equations 10, 11, 12 into the Lagrangian equation[38],
\[
\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \mathbf{q}} \right) - T \frac{\partial}{\partial \mathbf{q}} = Q_i \quad (i = 1, 2, \ldots n)
\]
and after some mathematical simplifications, the equation of motion for fluid domain can be written as,
\[
M_i \ddot{U} + K_i \ddot{U} + S_i \dot{U} = F_i
\]
Or
\[
M_i \ddot{U} + K_i \ddot{U} = F_i \quad \text{where} \quad K_i = K_i + S
\]
In equation 15, \(M_i\), \(K_i\) and \(F_i\) respectively express the mass matrix, stiffness matrix and force matrix of solid medium, respectively. In addition to equations 15 and 16, the coupled equation of motion for solid-fluid system can be obtained considering boundary conditions at the interface between solid and fluid medium. Therefore, the equation of motion for damped solid (structure and foundation) medium can be expressed as,
\[
M_i \ddot{U} + C_i \dot{U} + K_i U = F_i
\]

Where, \(M_i\), \(C_i\) and \(K_i\) express the mass matrix, damping matrix and stiffness matrix of the solid medium, respectively. In addition to equations 15 and 16, the coupled equation of motion for solid-fluid system can be obtained considering boundary conditions at the interface between solid and fluid medium. Therefore, the displacements perpendicular to the interface should be equal. This interface between solid and fluid medium. Therefore, the equation of motion for solid medium can be obtained considering boundary conditions at the interface between solid and fluid medium. Consequently, the equation of motion for damped solid-structure (structure and foundation) medium can be expressed as,
\[
M_i \ddot{U} + C_i \dot{U} + K_i U + S_i \dot{U} = F_i
\]

Or
\[
M_i \ddot{U} + K_i \ddot{U} = F_i \quad \text{where} \quad K_i = K_i + S
\]

In order to satisfy the interaction between solid and fluid regions, the stiffness of interaction element is found to be 240 and 315.55E+16, testing many conditions. The rotational constraint parameter and stiffness of interaction elements are selected by evaluating using Finite Element Method. The numerical models are evaluated under plane strain condition. The numerical models are written in FORTRAN 90 programming language. The validation and accuracy of the program is firstly controlled analyzing the numerical models presented in References [35, 37, 41]. Due to the limitation of space, the validation results are not presented in this paper. Then, the static and dynamic results of the models given in Figure 1, Figure 2 and Figure 3 are presented with details.

In this equation, the structure-water interaction problem is considered to be analyzed. In this model the foundation of structure and reservoir region is assumed rigid. The length of reservoir and near foundation region [23] is assumed to be three times length of the structure height.

The damping ratio of the system is considered to be 3%. The mathematical model of this problem is firstly prepared in the Laplace transform domain. Then, the time domain responses of the system are calculated using Durbin’s numerical inverse Laplace transform technique [36]. The stiffness and mass matrices of fluid and solid regions are respectively calculated using 2X2 reduced numerical integration method [21] and 3X3 numerical integration method. Also, the stiffness and mass matrices of solid infinite region are calculated using 12 point Newton-Cotes numerical integration method [35].

3. Numerical Examples

In this section, static and dynamic responses of three different models are evaluated using Finite Element Method. The numerical models are analyzed under plane strain condition. The numerical models are written in FORTRAN 90 programming language. The validation and accuracy of the program is firstly controlled analyzing the numerical models presented in References [35, 37, 41]. Due to the limitation of space, the validation results are not presented in this paper. Then, the static and dynamic results of the models given in Figure 1, Figure 2 and Figure 3 are presented with details.

In order to satisfy the interaction between solid and fluid regions, the displacements along the interface nodes have to be equal. Therefore, some rigid one-dimensional elements are embedded along the interface nodes with the length of 1/1000 m. The constrain parameter in fluid and the stiffness of interaction elements are selected by testing many conditions. The rotational constraint parameter and stiffness of interaction element is found to be 240 and 315.55E+16, respectively.

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The foundation material in Figure 2 and Figure 3 has been assumed as homogeneous, isotropic and viscoelastic medium. The mechanical properties of structure, water and foundation region are given in Table 1. In Figure 2, the foundation length and depth are assumed to be three times length of the structure height [23]. Boundary conditions of near foundation region in Figure 2 are assumed to be constrained in x and y directions while, that of the fluid region in Figure 1, 2, 3 are assumed to be constrained only in x direction. The dimensions, number of points and interface node numbers of each model are clearly given in Figure 1, 2, and 3. In this paper, only the horizontal and vertical displacement responses of point 1 are given.

Table 1. Material Properties of Numerical models

<table>
<thead>
<tr>
<th>Medium</th>
<th>Bulk Modulus (Pa)</th>
<th>Modulus of Elasticity (Pa)</th>
<th>Mass Density (kg/m³)</th>
<th>Poisson Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid</td>
<td>2.07E+9</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structure</td>
<td>-</td>
<td>3.18E+10</td>
<td>2400</td>
<td>0.2</td>
</tr>
<tr>
<td>Foundation</td>
<td>-</td>
<td>2.08E+8</td>
<td>2000</td>
<td>0.35</td>
</tr>
</tbody>
</table>

4. Results and Discussions

4.1. Statics

At the points 1 and 95 of the Models 1, 2, 3, the static loads 2242kN and 1242kN are applied in opposite x direction, respectively. The static displacement responses of vertically selected 13 nodes inside the structure for each model are given in Table 2, 3, 4. As seen from Tables, the x direction displacement responses are getting smaller near to rigid foundation. Also, the static displacement responses of the interface nodes between solid and fluid regions are given. As seen in the Tables 2, 3, 4, the interface node displacements are very close to be equal. Therefore, the dynamic interaction between these regions is clearly provided with Lagrangian approach.

Table 2. Model 1 static responses

<table>
<thead>
<tr>
<th>MODE</th>
<th>X Disp (m)</th>
<th>Y Disp (m)</th>
<th>INTERFACE NODES</th>
<th>INTERFACE NODE DISP (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.55157E-04</td>
<td>0.21505E-04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-1.82599E-04</td>
<td>1.32480E-05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>-8.78087E-05</td>
<td>2.38774E-05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>-1.66820E-05</td>
<td>1.95631E-05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-1.10887E-06</td>
<td>1.92739E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-2.37760E-06</td>
<td>1.92691E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>-5.32506E-07</td>
<td>6.86939E-07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>108</td>
<td>-4.92650E-08</td>
<td>1.64031E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>109</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>187.188</td>
<td>0.0000000001</td>
</tr>
</tbody>
</table>

As seen in Table 2, the interface node displacements are too closed between two nodes.

The maximum horizontal static displacement of the structure is seen in node 1 of the structure. The rotation constraint parameter and stiffness of interaction element which are found by solving many conditions are required in dynamic analysis of the models again. The rotation constraint parameter 240 and stiffness of interaction element 315.55E+16 are utilized in transient analysis of the models.

4.2. Dynamic Responses

In the dynamic investigation of the models, horizontal ground acceleration data given in Reference [35] is imposed on the models as dynamic load.

The responses are drawn in displacement-time variation. In this investigation, the structure-foundation interaction problem is firstly analyzed and the responses for node 1 are given in Figure 2. Then, water region is added to the models. The effects of water and water compressibility considering bulk modulus of water on the models are seen to be very significant. The displacements are also increased when water is added to the models (structure-water, structure-water-foundation model).
Also, the bounded foundation region effect on the dynamic responses of the Model 2 is not giving actual behavior of the model. When the foundation is assumed to be bounded, the radiation of waves can’t happen at the boundaries. Therefore, the reflected waves come back to the structure. And, the actual behavior of the problem can’t be expected using bounded regions in foundation and water regions.

It has been seen that the responses of the structure in structure-foundation problem under horizontal ground motion was underestimated assuming the foundation rigid or bounded elastic region and unbounded elastic region is found to be 55.71%. Likewise, in structure-water-foundation interaction problem, the dynamic responses of structure-water-foundation problem considering spatially-varying ground motion was underestimated assuming the foundation rigid or bounded region.

In order to see the effects of far end boundaries in the water domain, further research is needed.

**5. Conclusions**

In the presented study, visco-elastic behavior of coupled structure-water-foundation interaction problem is investigated solving three examples. As demonstrated by three examples, excellent results have been obtained considering the compressibility effects of water and using infinite elements in the far end boundaries of the foundation. And these two parameters significantly affect the dynamic responses of structure-water-foundation problem.

Figure 5. The structure-foundation interaction problem for Models 1, 2, and Model 3

Figure 6. The structure-water-foundation interaction problem for Models 1, 2, and Model 3

References


Nomenclature

\[ \rho_f : \text{The mass density of the fluid} \]
\[ g : \text{Gravitational acceleration} \]
\[ \zeta : \text{Damping ratio of the system} \]
\[ s : \text{Laplace complex variable} \]
\[ W : \text{Rotational equation} \]
\[ K : \text{The Laplace transform of stiffness matrix} \]
\[ \pi : \text{The total potential energy of fluid domain} \]


