



### A New Equation for Analytical Prediction of Shear Capacity of Steel Fiber Reinforced Concrete Deep Beams

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#### Abstract

The design of many structural elements requires the use of deep concrete beams. However, their geometrical proportions make these beams viable to total shear failure without ample warning. Available researches have shown that the employment of steel fibers in concrete improves the structural performance of deep beams. However, validated capacity prediction models that facilitate the design for these beams are scarce in the approachable references. Several investigators have been suggesting predictive formulae for the evaluation of the shear strength of steel fiber reinforced concrete (SFRC) beams. In the current study, an equation that can predict the shear capacity of SFRC deep beams was derived in terms of compressive strength of concrete, shear span, beam sectional dimensions, and fiber factor. The derived equation compares fairly well with the authors' experimental data and the data available in the literature.

#### 1. Introduction

At present, the architectural design requires sizeable free space for many multi-story buildings, which mandates the use of deep beams as load transfer mediums. In addition, various structural elements can be classified as "deep beams", such as laterally loaded slabs, shear walls, and pile caps [1]. There is no unique definition for a deep/shallow beam. The ACI Committee 318 [2] defines the deep beam as that having a clear span-to-depth ratio of less than four ( $L/d < 4$ ) and shear-span to depth ratio ( $a/d < 2$ ). However, ACI-ASCE Committee 426 [3] classifies a beam having ( $a/d < 1$ ) as "deep beam", and the beam having ( $a/d > 2.5$ ) as "an ordinary or shallow beam". Any beam in between these two limits is categorized as a "moderate deep beam". Moderate deep and deep beams are predominant shear-deficient (i.e. having insufficient web reinforcement) members and generally fail in brittle or catastrophic shear mode [4].

For the last three decades, research efforts have been devoted to investigating experimentally and numerically the influence of steel fibers on the structural behavior of RC deep beams. Various researchers [5-11] have reported that the load capacity, resistance to spalling, and ductility of the beams, with or without openings, subjected to monotonic loading increases as the content of the fiber increases and  $a/d$  decreases. This favorable effect of steel fibers has been attributed to their bridging actions across main and secondary cracks [12]. It is worth noting that there is a general agreement between researchers that the strut-tie-model is an appropriate method for simulating the mechanical performance of deep beams [13, 14].

The review of the literature shows that the design of many structural elements requires the use of deep concrete beams. However, their geometrical proportions make these beams viable to total shear failure without ample warning (brittle failure). Available researches have shown that the employment of steel fibers in concrete improves the structural performance of deep beams. However, the influence of

steel fibers in improving the deformability and capacity of deep beams against shear failure is not studied much in the approachable references.

In the present study, the popular existing predictive formulas are first reviewed, and then a new equation is proposed for predicting the shear capacity of SFRC deep beams in terms of major design variables such as compressive strength of concrete, shear span, beam sectional dimensions, and fiber factor. The proposed equation is compared with the other existing equations and the data generated in our funded project [15].

#### 2. Existing Predictive Formulas

Since 1970, various researchers have been proposing predictive models for the evaluation of the shear strength of steel fiber reinforced concrete (SFRC) beams. Most of these models include the fiber factor ( $F$ ), which combines the fiber aspect ratio and volume, and  $a/d$  ratio in shear strength prediction model. There is a wide diversity in the results of the predicted shear strength of SFRC beams. Slater et al. [8] and Arslan [9] have presented a comprehensive review of the available predictive models for the ultimate shear strength of SFRC beams. Most of these models were developed based on 1) linear/nonlinear regression analysis of experimental results, 2) modification of the ACI code model, 3) genetic algorithms, and 4) analytical approach using the principles of mechanics. Some of the predictive equations based on the principles of mechanics are presented in [9, 10, 16-18]. However, the semi-empirical-based models could be retrieved from [19-24]. It is worth noting that some of these predictive models are available for high strength [18, 22] and lightweight fiber reinforced concrete beams [17]. Out of these models, the ones proposed by ACI 318 [2] and Khuntia et al. [18] are employed to compare with the model of the proposed study.

##### 2.1 ACI 318 [2]

Section 9.9.2.1 of ACI 318-2014 [2] proposes the following simple shear strength equation for deep beams:

$$V_u = 0.83\sqrt{f'_c} b_w d \quad (1)$$

where,  $f'_c$  is the compressive strength of concrete (MPa),  $b_w$  is the width of the beam (mm), and  $d$  is the effective depth of the beam (mm).

The above conservative ACI 318 equation is essentially proposed for selecting the dimensions of the deep beams. The effect of fibers was neglected in this expression.

## 2.2 Khuntia et al. model[18]

In the model of Khuntia et al. [18], the contribution of the compression region, aggregate interlock, and dowel action were lumped into a single term:

$$V_u = [0.167\alpha + 0.25F\sqrt{f'_c}] b_w d \quad (2)$$

In the above equation,

$$\alpha = 1 \text{ for } \frac{a}{d} \geq 2.5 \quad (3)$$

$$\alpha = 2.5 \frac{d}{a} \text{ for } \frac{a}{d} < 2.5 \quad (4)$$

where,  $f'_c$  is compressive strength of concrete (MPa),  $F$  is fiber factor,  $b_w$  is width of the beam (mm),  $d$  is effective depth of the beam (mm), and  $a$  is shear span (mm).

The expression of Khuntia et al. [18] was validated against tested beams with a wide range of variables and was shown to be conservative.

## 3. Proposed Analytical model

In the current research, a data set was prepared to contain the shear strength test results of control and SFRC deep beams available in the literature, and that tested in our funded project supported by the Deanship of Scientific Research, King Saud University, Riyadh, Saudi Arabia. Eq.(5) was then derived for estimating the shear capacity of control and SFRC deep beams. The equation was obtained by minimizing the error between the predicted and experimentally observed shear strength values.

$$V_u = 0.41 \sqrt{f'_{cf}} \left( \frac{h}{a} + \frac{V_f \alpha}{h} \right) \exp(\alpha \sqrt{RI}) b_w d \quad (5)$$

Where,

$$RI = \text{reinforcing index} = V_f L_f / d_f \quad (6)$$

$\alpha = 0.018$  (an empirical constant)

It should be noted that the compressive strength of SFRC ( $f'_{cf}$ ) was estimated using the model proposed by Abadel et al. [25] as given by Eq.(7):

$$f'_{cf} = f'_c + 5.222(RI) \quad (7)$$

In these expressions,  $f'_c$  is the compressive strength of concrete (MPa),  $f'_{cf}$  is the compressive strength of SFRC (MPa),  $h$  is the overall depth of the beam (mm),  $V_f$  is the volume fraction of fiber (in %),  $L_f$  is the length of fiber (mm),  $d_f$  is the diameter of fiber (mm),  $b_w$  is the width of the beam (mm),  $d$  is the effective depth of the beam (mm), and  $a$  is the shear span (mm).

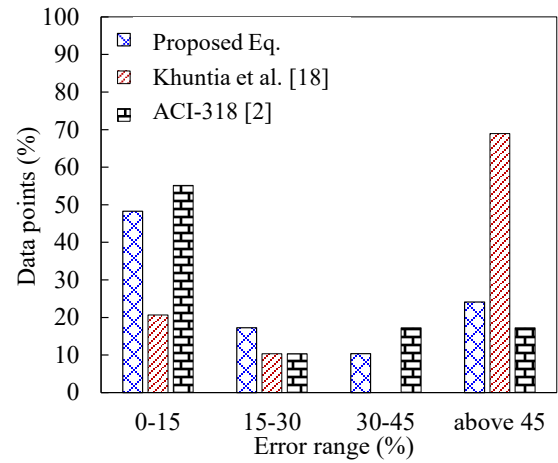


Figure 1. Error analysis for various prediction models

## 4. Discussion of Results

The performance of the prediction model (Eq. 5) is presented in Table 1 and summarized in Figure 1. A total of nine data sets were used to show the performance of the proposed equation. Eight data sets were taken from the literature, and one data set was that of the authors themselves [15]. Table 1 clearly shows that the proposed model is performing very well for all nine data sets. The ratio of predicted versus experimentally observed values is fairly close to the unity. The average ratio is, in fact, 1.0 with the standard deviation = 0.41.

The proposed model also compares fairly well with the ACI 318 and Khuntia et al. [18] models. Figure 1 clearly shows that more than 50% of the data predict the experimentally observed values very well, as the error in these predictions is less than or equals to 15%. Figure 1 also illustrates that the proposed model performs better than the ACI 318 [2] model and Khuntia et al. [18] models. This concludes that the proposed equation can be employed to estimate the nominal shear capacity of the control and SFRC deep beams with reasonable accuracy.

Table 1. Observed vs. predicted ultimate load for various SFRC beams

Ref	Beam	$a/d$	$b$ (mm)	$d$ (mm)	$f'_c$ (MPa)	$V_f$ (%)	Ultimate Load (kN)				Error (%)			Pred/Exp					
							Observed	Predicted			Proposed Eq.	Khuntia et al. [18]	ACI-318 [2]	Proposed Eq.	Khuntia et al. [18]	ACI-318 [2]	Proposed Eq.	Khuntia et al. [18]	ACI-318 [2]
								Proposed Eq.	Khuntia et al. [18]	ACI-318 [2]									
Authors	Beam-1	0.6	200	520	38.9	0.00	1247.2	1063.8	1083.2	1076.8	14.70	13.14	13.66	0.85	0.87	0.86			
	Beam-2	0.6	200	520	38.9	0.00	1248.2	1063.8	1083.2	1076.8	14.77	13.21	13.73	0.85	0.87	0.86			
	Beam-3	0.6	200	520	42.1	0.50	1383.1	1378.0	1234.2	1120.2	0.36	10.76	19.01	1.00	0.89	0.81			
	Beam-4	0.6	200	520	42.1	0.50	1384.1	1378.0	1234.2	1120.2	0.44	10.83	19.07	1.00	0.89	0.81			
	Beam-5	0.6	200	520	46.2	1.00	1467.6	1672.8	1405.3	1173.4	13.98	4.25	20.05	1.14	0.96	0.80			
	Beam-6	0.6	200	520	46.2	1.00	1468.6	1672.8	1405.3	1173.4	13.90	4.31	20.10	1.14	0.96	0.80			
	Beam-7	0.6	200	520	50.9	2.00	1342.1	2236.1	1711.0	1231.7	66.61	27.48	8.23	1.67	1.27	0.92			
	Beam-8	0.6	200	520	50.9	2.00	1343.1	2236.1	1711.0	1231.7	66.48	27.39	8.30	1.66	1.27	0.92			
[26]	FC2	1.6	152	558	54.1	0.75	549.6	526.7	233.0	517.8	4.17	57.61	5.79	0.96	0.42	0.94			
	FC3	1.6	152	558	49.9	1.50	646.3	881.5	291.1	497.3	36.38	54.95	23.06	1.36	0.45	0.77			
	FC11	1.6	152	558	40.8	0.60	473.3	404.7	222.6	449.7	14.48	52.96	4.99	0.86	0.47	0.95			
[22]	B-1-0.5-A	1.0	125	215	99.0	0.50	244.3	181.5	131.7	221.9	25.71	46.09	9.15	0.74	0.54	0.91			
	B-1-1.0-A	1.0	125	215	95.3	1.00	342.4	247.3	148.9	217.8	27.77	56.51	36.40	0.72	0.43	0.64			
	B-1-1.5-A	1.0	125	215	96.4	1.50	374.9	320.8	169.5	219.0	14.42	54.78	41.58	0.86	0.45	0.58			
[19]	B10	2.0	150	219	40.9	1.00	230.0	247.6	75.4	174.4	7.66	67.22	24.17	1.08	0.33	0.76			
	B20	2.0	150	219	43.2	2.00	231.3	485.2	109.8	179.2	109.80	52.50	22.51	2.10	0.47	0.77			
	A10	2.0	150	219	40.9	1.00	192.5	247.6	75.4	174.4	28.60	60.85	9.42	1.29	0.39	0.91			
[10]	4/0.5/1.5	1.5	152	221	34.0	0.50	268.7	125.6	69.2	162.6	53.28	74.25	39.50	0.47	0.26	0.60			
	4/1.0/1.5	1.5	152	221	34.0	1.00	294.3	200.0	83.9	162.6	32.02	71.49	44.75	0.68	0.29	0.55			
[27]	2.2/2	1.5	200	260	41.2	0.25	559.5	153.4	106.9	277.0	72.58	80.90	50.49	0.27	0.19	0.50			
	2.2/3	1.5	200	260	40.3	0.76	599.0	278.0	133.9	274.0	53.59	77.65	54.26	0.46	0.22	0.46			
[20]	S3F	1.8	150	267	48.6	0.90	235.5	149.9	69.2	231.7	36.34	70.61	1.59	0.64	0.29	0.98			
[17]	1TLF-1	2.0	55	265	35.6	1.00	159.7	291.0	116.9	72.2	82.15	26.82	54.82	1.82	0.73	0.45			
	2TLF-1	2.0	55	265	37.8	1.00	143.1	106.7	39.9	74.4	25.44	72.13	48.03	0.75	0.28	0.52			
	3TLF-1	2.0	55	265	35.7	1.00	135.0	110.0	41.1	72.3	18.52	69.54	46.44	0.81	0.30	0.54			
[16]	2	2.0	60	340	35.0	0.50	108.9	106.9	40.0	100.2	1.90	63.33	8.05	0.98	0.37	0.92			
	3	2.0	60	340	33.0	0.75	90.4	81.9	34.2	97.3	9.36	62.11	7.63	0.91	0.38	1.08			
	4	2.0	60	340	36.0	1.00	105.1	108.4	37.6	101.6	3.21	64.17	3.30	1.03	0.36	0.97			
	6	1.5	60	340	36.0	1.00	153.4	144.2	43.9	101.6	5.98	71.38	33.78	0.94	0.29	0.66			
Mean										29.47	48.94	23.86	1.00	0.55	0.77				
Standard deviation										27.91	24.73	17.16	0.41	0.31	0.18				

## 5. Conclusions

In the present study, an equation that can predict the nominal shear capacity of SFRC deep beams was derived in terms of the compressive strength of concrete  $f'_c$ , shear span  $a$ , beam sectional dimensions ( $b_w, d, h$ ) and reinforcing index  $RI$ . This equation compares well with the authors' experimental data and the data available in the literature.

## Declaration of Conflict of Interests

The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Nomenclature

$F$  : fiber factor  
 $f'_c$  : compressive strength of concrete (MPa)  
 $f'_{cf}$  : compressive strength of SFRC (MPa)  
 $b_w$  : width of the beam (mm)  
 $h$  : overall depth of the beam (mm)  
 $a$  : shear span (mm)  
 $d$  : effective depth of the beam (mm)  
 $RI$  : reinforcing index =  $V_f L_f / d_f$   
 $V_f$  : volume fraction of fiber (in %)  
 $L_f$  : length of fiber (mm)  
 $d_f$  : diameter of fiber (mm)  
 $\alpha$  : an empirical constant

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