PACE-2021
International Congress on the Phenomenological Aspects of Civil Engineering
Research Article 20-23 June 2021

Dependence Of Partial Factors For Actions To The Reliability Index

Lazar Živković*, Biljana Matejević-Nikolić, Jovan Ristić, Nemanja Marinković
University of Niš, Faculty of Civil Engineering and Architecture, Serbia
Corresponding Author E-mail:lazar.zivkovic@gaf.ni.ac.rs

Keywords

Reliability,
Reliability Index,
Partial Factors,
Ultimate Limit State,
Service-Ability Limit State.

Abstract

This paper presents the procedure for determining partial factors for actions depending on the reliability index. Structural reliability classes and the concept of reliability index are described. Based on the analytical terms from EN 1990, the partial coefficients for actions were calculated. The dependence of the partial factors for permanent and variable actions and the reliability index for different coefficients of variation when the load capacity is normally and distributed according to Gumbel is presented.

1.Introduction

The reliability of a structure is the ability of the structure, or its element, to meet the prescribed requirements, including the prescribed service life, for which they have been designed. In quantitative terms, reliability can be defined as the complement to the probability of failure [1]. Reliability is expressed in probabilistic terms. It covers the safety, serviceability (functionality) and durability of the structure (Figure 1) [2].

Safety - a structure with an appropriate degree of safety must be able to sustain all loads that will occur during its life (load-bearing capacity, stability control, ...) – Ultimate Limit State (ULS);

Functionality - the structure must facilitate the normal functioning of the building according to its purpose (deformation and vibration control) – Service-ability Limit State (SLS);

Durability - factors of influence are choice and quality of materials, quality of construction works, construction design, level of maintenance of the facility [3].

EN 1990 [4] sets out the principles and requirements for the safety, serviceability and durability of structures. The basics for their calculation with budgetary evidence are provided and instructions for the relevant aspects of structural reliability are presented.

Different levels of reliability can be adopted, among other things, for the load-bearing capacity of the structure and serviceability. The choice of a reliability level, for the observed construction, should take into account the relevant factors, which include:

- possible cause and/or manner of reaching the limit state,
- possible consequences of a failure, which would endanger human lives and economic losses,
- public aversion to failure,
- necessary costs and procedures to reduce the risk of failure [4].

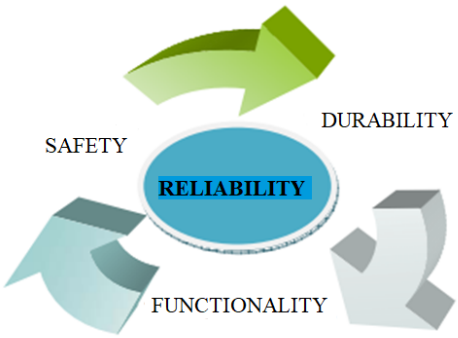

Figure 1. Figure 1 – Definition of reliability according to [2]

The levels of reliability, which should be applied to a particular structure, can be prescribed through the classification of the structure as a whole and/or the classification of its elements.

The classification according to reliability can be represented by reliability indices β , which serve to take into account the adopted or assumed statistical variability of the impacts and bearing capacity actions, as well as the unreliability of the model.

2. Structure categories according to the failure effects

Based on the consideration of the effects on human life and health, as well as the economic factors that cause the collapse of the structure or the poor functioning of the structure, according to [4], classes according to the effects of CC were introduced.

Table 1. Definition of classes according to effects

Class according to effects	Description	Examples of buildings and other structures	Parameter ρ
CC3	Severe effects on the loss of human lives, or very severe economic social environmental effects	Spectator stands, public buildings, where the effects of the failure are considerable (eg concert hall)	$5 \leq \rho < 10$
CC2	Medium effects on the loss of human lives, considerable economic, social, or environmental effects	Housing and administrative buildings, public buildings with medium failure effects (eg. Administrative buildings, industrial buildings)	$2 \leq \rho < 5$
CC1	Minor effects on the loss of human lives, or minor economic social environmental effects	Agricultural buildings not normally visited by people (storage facilities, glasshouses, silos, masts	$2 < \rho$

According to [6] the classes according to the effects are determined based on an appropriate number of people who are at risk due to structural failure. The classes are shown in Table 1. According to the effects, three reliability classes RC1, RC2 and RC3 can be defined via the reliability index β (Table 2).

Table 2. Recommended minimal values for the β reliability index (ultimate limit states), according to [4]

Class according to reliability	Minimal values for reliability index β	
	The reference period of 1 year	The reference period of 50 years
RC3	5.2	4.3
RC2	4.7	3.8
RC1	4.2	3.3

There are also structures, or structural elements (eg lighting poles, mast, etc.), which are classified in category RC1 for economic reasons, but such structures are subject to stricter levels of design revision and supervision.

Another type of division of structures into classes according to the effects of failure can be based on determining the parameter ρ which according to [5] is defined as the ratio of the total cost of construction (sum of construction costs and costs of its removal due to failure) and construction costs (Table 1).

3. Reliability index

The proof of structural reliability which is most often implemented in practice is conducted using the operative value of failure probability P_f which is called the reliability index β . The proof of reliability in this case according to [2] is:

$$\beta_{cal} \geq \beta_{req} \tag{1}$$

where:

β_{cal} - is the reliability index, calculated for a structure,

β_{req} - is the reliability index which is norm adjusted for a specific class of structural failure effects (table 2).

Failure probability can be expressed through the reliability index β defined by the expression:

$$\beta = -\Phi_U^{-1}(P_f) \tag{2}$$

where:

Φ_U^{-1} - is an inverse standardized function of normal distribution.

Table 3. Dependence of failure probability and reliability index according to [1]

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.2

Failure probability and reliability index are used to quantify the same thing – reliability of the structure [1]. Table 3 shows the connection between β and P_f . Also, for all types of structures, the reliability class RC2 and service life of 50 years are provided. This refers to „ordinary" structures, because of the reliability indices β = 3,8 (ultimate limit state) and β=1,5 (service-ability limit state) partial factors are recommended in the norm. In the case of „extraordinary" structures which are classified as RC3, the reliability index β = 4,3 (ULS) is adopted. The code does not define which values of partial factors are to be adopted for the design of such structures [2].

4. Partial safety coefficients

4.1. Reliability methods

By using partial safety coefficients and ψ coefficients for basic variables, such as actions, geometrical properties of an element, etc, in the partial coefficient methods are obtained the design values. It is also necessary to conduct a calculation proof to prove that no limit state (ULS and SLS) has been exceeded.

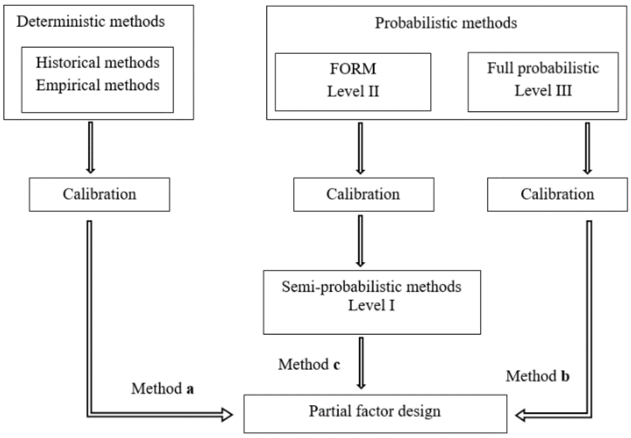

Figure 2. Review of the reliability methods, according to [4]

Numerical values of partial factors and ψ coefficients according to [4] can be determined:

- Based on the calibration (based on experience in construction, the leading approach in Eurocodes) and

- Field observations and statistical estimates of experimental research (the levels of reliability for representative constructions must be as close as possible to the target reliability index).

Based on the scheme in Figure 2, probabilistic calibration procedures can be divided into:

- Fully probabilistic methods - level III (rarely used in regulations for the calculation of structures),

- First-order reliability methods FORM - level II (accurate enough for most applications on structures).

Eurocodes (EN 1990 to EN 1999) was originally based on method A. The probability that the structure will not fail, with which a certain measure of reliability should be determined, is represented by the expression:

$$P_s = 1 - P_f \tag{3}$$

where:

P_f - is the probability of structural failure.

Therefore, based on the expression (3) it can be considered that the value P_s is actually the structural reliability. In engineering practice, it is common to look for the value of failure probability [2].

4.2. Calculation point of the ultimate state function

The operational application of the proof of reliability with Eurocode is based on the method of partial factors. Theoretical bases of reliability engineering are used to determine partial factors γ_i, reduction coefficients ψ_i, rules for combined actions, etc. The failure probability P_f can be presented using expression (4) via the behavior function *g*. If $g > 0$ the structure will survive, and if $g \leq 0$ it will fail, whereby the failure point is defined for $g = 0$.

$$g = R - E \tag{4}$$

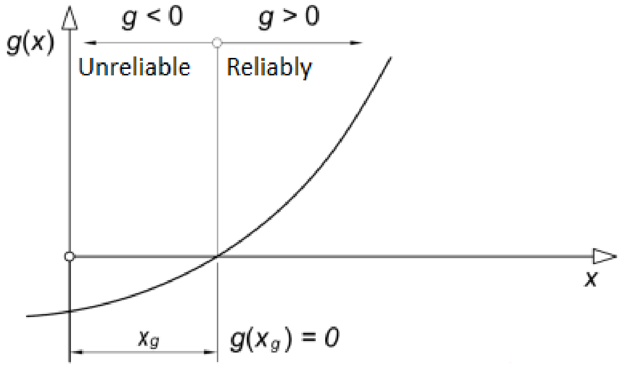

Figure 3. Behavior function g, according to [3]

Figure 4 shows the function of the limit state *R* - *E* = 0 by two random variables, namely the influence of the action *E* and the bearing capacity *R*. Accordingly, it follows that the failure limit is *R=E*. Furthermore, based on that, it follows that $\sigma_R = \sigma_E$ that is, that the standard deviation of the bearing capacity and the standard deviation of the effects are from actions of the same magnitude.

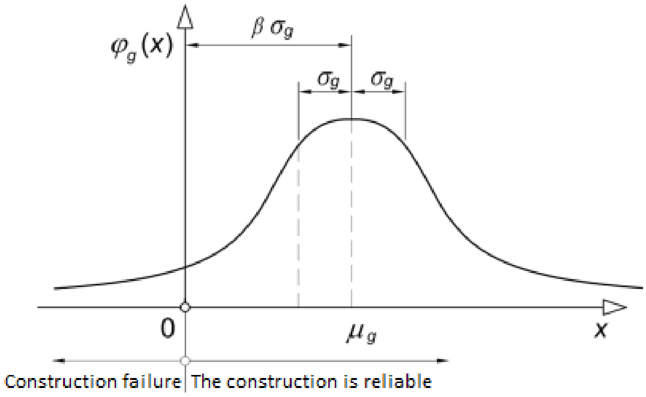

Figure 4. State of structural safety, according to [3]

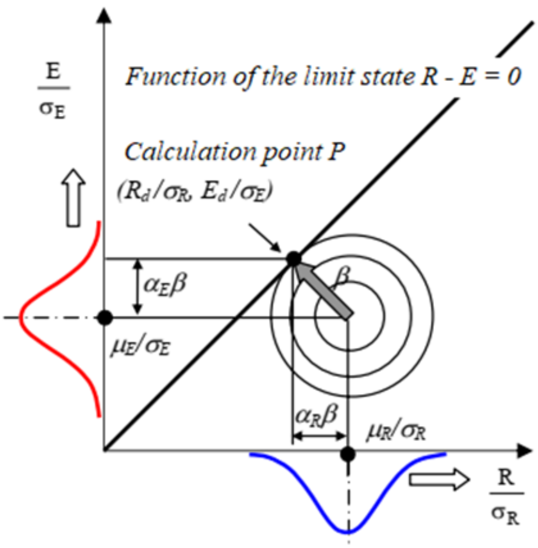

Figure 5. Theoretical basics of reliability engineering for determination of calculation point, [2]

Figure 5 shows that the reliability zone is below the failure limit. This is not a good starting point for proof of reliability. In the case of proof of structural reliability, theoretically, each point is in the direction of the failure limit $R - E = 0$. However, for the proof of reliability, the point at which the probability of failure is the highest is adopted. This point has coordinates $P(\frac{R_d}{\sigma_R}, \frac{E_d}{\sigma_E})$, and in reliability theory, it is called a calculation point. If it is:

$$\beta = \frac{\mu_g}{\sigma_g} \tag{5}$$

where:

μ_g - is the mean value for *g*,

σ_g - is its standard deviation,

So that:

$$\mu_g - \beta \cdot \sigma_g = 0 \tag{6}$$

From figure 5, coordinates of the calculation point can be written in the following form:

$$\frac{R_d}{\sigma_R} = \frac{\mu_R}{\sigma_R} - \alpha_R \cdot \beta; \Rightarrow R_d = \mu_R - \alpha_R \cdot \beta \cdot \sigma_R \text{ , and}$$

$$\frac{E_d}{\sigma_E} = \frac{\mu_E}{\sigma_E} - \alpha_E \cdot \beta; \Rightarrow E_d = \mu_E - \alpha_E \cdot \beta \cdot \sigma_E \tag{7}$$

Where:

μ_R - is the mean bearing capacity value,

μ_E - is the mean action effect value,

α_R - is the vulnerability coefficient, for action effects, by the first-order reliability method FORM, ($\alpha_R = 0{,}8$)

α_E - is the vulnerability coefficient, for bearing capacity, by the first-order reliability method FORM

($\alpha_E = -0{,}7$).

The values for α_R and α_E are valid only inside the range $0{,}16 < \frac{\sigma_E}{\sigma_R} < 7{,}6$.

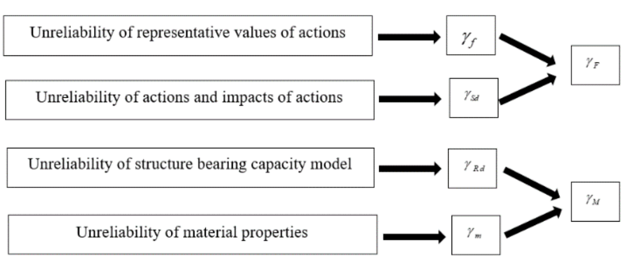

Figure 6. Relation between individual partial factors, according to [4]

4.3. Partial factors for actions

a) Partial factors for permanent actions

Partial factor for permanent actions are represented by the expression:

$$\gamma_G = \frac{G_d}{G_k} \tag{8}$$

Where is:

G_d - design values of permanent actions,

G_k - characteristic values of permanent actions.

Towards [2] it is usually accepted that the characteristic value of the constant action G_k is equal to the mean value μ_G, so it is:

$$\gamma_G = \frac{\mu_G \cdot (1 - \alpha_E \cdot \beta \cdot V_G)}{\mu_G} \tag{9}$$

Where is:

μ_G - mean value of permanent action,

V_G - coefficient of variation of total permanent action,

α_E - sensitivity factor of for effects of actions,

β - reliability index.

Expression (9) can therefore be written in the form:

$$\gamma_G = 1 - \alpha_E \cdot \beta \cdot V_G \tag{10}$$

Coefficient of variation V_G obtained by expression:

$$V_G = \sqrt{V_g^{\,2} + V_{Ed}^{\,2}} \tag{11}$$

Gde je:

V_g - coefficient of variation of permanent action,

V_{Ed} - coefficient of variation of design performance model permanent action.

The present values in [2] are $V_g = 0{,}10$ and $V_g = 0{,}10$ so it is:

$$V_G = \sqrt{0{,}10^2 + 0{,}10^2} = 0{,}14$$

According to the expression (9) the partial safety factor for permanent action is then:

$$\gamma_G = 1 - (-0{,}7) \cdot 3{,}8 \cdot 0{,}14 = 1 + 0{,}7 \cdot 3{,}8 \cdot 0{,}14 = 1{,}37 \cong 1{,}35.$$

This value of γ_G is adopted in Eurocode.

Figure 7 shows the dependence of the reliability index and the partial factor of permanent action γ_G, for the coefficient of variation applied in Eurocode.

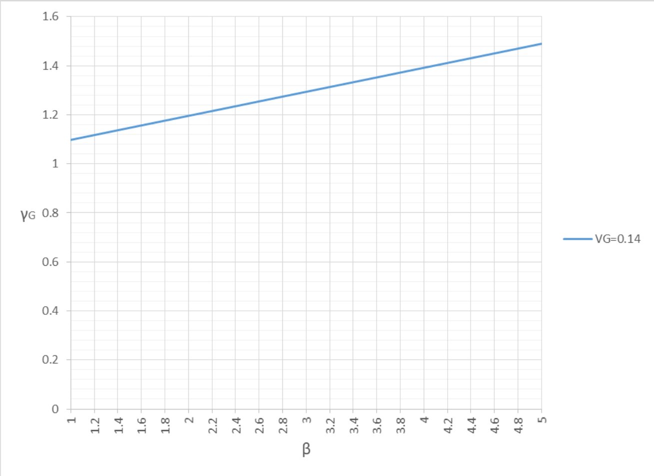

Figure 7. Dependence of partial factors for the permanent action γ_G and reliability index β for the coefficient of variation $V_G = 0,14$, for normal distribution

b) Partial factors for variable actions

Partial factor for variable actions are represented by the expression:

$$\gamma_Q = \frac{Q_d}{Q_k} \tag{12}$$

Where is:

Q_d - design values of variable actions,

Q_k - characteristic values of variable actions.

If, on the basis of the present in [2], the distribution function of the variable fact adopt the Gumbel distribution and the 98% fractile, provided the values of the partial factors for variable action according to the formula:

$$\gamma_Q = \frac{\mu_Q \cdot (1 - V_Q \cdot (0,45 + 0,78\ln(-\ln(\phi(-\alpha_E \cdot \beta)))))}{\mu_Q \cdot (1 - V_Q \cdot (0,45 + 0,78\ln(-\ln(0,98))))} \tag{13}$$

Where is:

μ_Q - mean value of variable action,

V_Q - coefficient of variation of variable action,

ϕ - cumulative distribution function of the standardized Normal distribution,

α_E - sensitivity factor of for effects of actions,

β - reliability index.

Figure 8 presented the dependence of partial factors for the variable action γ_Q and reliability index β for the different coefficient of variation V_Q.

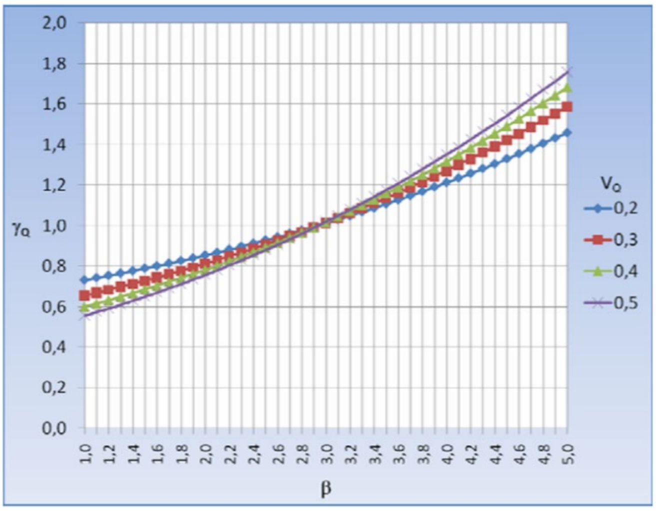

Figure 8. Dependence of partial factors for the variable action γ_Q and reliability index β for the different coefficient of variation V_Q, for distribution according to Gumbel, [2]

It can be seen from Figure 8 that for the coefficient of variation $V_Q = 0,5$ partial factors for variable actions are $\gamma_Q = 1,3$. However, the value V_Q can be greater than 0.5, and some other distribution function can better approximate the histogram of variable action [2].

5. Conclusion

The paper briefly presents the concept of structural reliability and refers to the control of load-bearing capacity, serviceability, and durability of the structure. Therefore, different reliability modalities can be defined for different categories of structures in different ways. One of the most convenient ways to represent reliability is through the reliability index β .

EN 1990 establishes principles and requirements in terms of safety and reliability of structures used on all other Eurocodes. Their assessment of reliability and durability is based on the principle of ultimate states in combination with safety coefficients.

This paper presents the procedures and expressions that define the safety coefficients in Eurocode. An unavoidable parameter in determining them is certainly the reliability index. In Eurocodes, the target value $\beta = 3,8$ of the reliability index for common constructions was used for safety factors. Conventional constructions, regardless of the material and static system, are classified in reliability class 2 (RC2). Also, the paper analyzes the relationship between different values of the reliability index and safety coefficients for the actions. For the needs of the analysis of the ratio of partial factors and reliability indices, the adopted coefficients of variation and sensitivity coefficients α_E and α_R were used.

Declaration of Conflict of Interests

The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1.] Marinković, S., Ignjatović, I.. Savremeni koncept obezbeđivanja trajnosti betonskih konstrukcija – projektovanje prema upotrebnom veku. IKS, Beograd (2013).

[2.] Andorić, B., Dujmović, D., Lukčević, I.. Razlike u procjeni pouzdanosti uobičajenih i iznimnih konstrukcija. Građevinar 61 (10) (2009) 943-953.

[3.] Marković, Z.. Osnove proračuna čeličnih konstrukcija - materijal sa predavanja. Građevinski fakultet, Univerzitet u Beogradu, Beograd (2018).

[4.] European Committee for Standardization (CEN). EN 1990:2002, Evrokod 0: Osnove proračuna konstrukcija. Građevinski fakultet, Univerzitet u Beogradu (2006).

[5.] Joint Committee of Structural Safety (JCSS). Probabilistic Model Code, Part 1 – Basis of Design. Swiss Federal Institute of Technology, Zurich (2001).

[6.] ASCE standard ASCE/SEI 7-10. Minimum design loads for buildings and other structures. American Society of Civil Engineers (2010).

[7.] Leonardo da Vinci Pilot Project. Handbook 2: Reliability Backgrounds, Prague (2005).

[8.] Holicky, M., Diamantidis, D., Sykora M.. Determination of Target Safety for Structures. 12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP 12, Vancouver, Canada (2015).

[9.] Stanojev, M., Stojić, D.. Reliability analysis of structures. Facta Universitatis, Series: Architecture and Civil Engineering 12 (3) (2014) 265-272.