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The Vibration of the FGM Beam Affected by Mass Damper

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Abstract

The effect of the mass damper on the fundamental frequency reduction of the functionally graded material (FGM) beam is investigated. Mori-Tanaka homogenization scheme is used to model through-the-thickness material gradation. The various classical boundary conditions, including simply supported beam, clamped beam, pinned-clamped beam and cantilever beam as well as shear hinge end condition are considered. It is assumed that the lumped mass damper is connected to the beam at an arbitrary position without sliding. The total potential energy is minimized by employing spectral Ritz method to calculate fundamental frequency and corresponding mode shape. The reduction of the frequencies in the presence of the attached lumped mass damper is observed. The dimensionless frequency reduction is affected by lumped mass amount and position. The lumped mass position plays an important role for vibration control of the beam.

1. Introduction

For passive vibration control, the nonlinear vibration absorbers are studied thoroughly. The exact nonlinear dynamics of a simply-supported beam carrying a nonlinear spring-inerter-damper energy absorber for primary resonance vibration reduction is studied. To this purpose, the analytical model and nonlinear dynamic responses are derived for a beam-spring-inerter-damper system [1]. The effects of sand size and volume fraction on attenuation of a beam free vibration is investigated. A high-speed camera was used to measure the damping ratio of the beam vibration. It is observed that the damping ratios for partially filled beams is greater than completely filled and empty beams. Also, it is found that the maximum damping ratio occurred at different mass fractions [2]. The vibration control of a rotating functionally gradient material (FGM) beam under thermal environment by using an enhanced active constrained layer damping (EACL D) treatment is studied. The mentioned model is extended from a newly developed dynamic model for composite EACL D beams. The difference is that the base beam is composed of temperature-dependent FGM that was widely used in the aerospace industry as advanced heat-resisting composite and the temperature dependence of viscoelastic material in the constrained layer is also considered [3]. The influence of the boundary relaxation on the free vibration characteristics of a rotating composite laminated Timoshenko beam is studied. Based on the first order shear theory, the theoretical model of the rotating composite laminated Timoshenko beam is established, in which the Possion's effect is also considered. The relaxed boundary conditions of the beam are simulated using a set of artificial springs. By adjusting the stiffness of the springs different extent relaxation boundary conditions of the beam can be obtained. Relaxation parameters are introduced to evaluate the extent of boundary relaxation. A uniformed formula for the centrifugal force of the rotating beam with relaxed boundary conditions is deduced [4]. The vibration characteristics of a cantilever L-shaped beam perpendicular to the horizontal plane are investigated. The dynamical model of the L-shaped multi-beam structure is established by using the Euler-Bernoulli beam theory, where vibration equations of each substructure, matching conditions

of the connection and boundary conditions are included. Based on the modal orthogonality, the discrete dynamical model with finite degrees of freedom is obtained. Then the distributed piezoelectric energy harvester is proposed to collect the vibration energy of the L-shaped beam [5].

In present work, the effect of the lumped mass damper position on the fundamental frequency reduction of the FGM beam is investigated. The volume fraction distribution of the functionally graded material is used to model mass density. The lumped mass is connected to the beam at an arbitrary position without sliding. The FGM beam has a small thickness to length ratio, consequently the total potential energy is derived based on Euler-Bernoulli beam theory. The total potential energy is minimized by employing spectral Ritz method to calculate fundamental frequency and corresponding mode shape. The various classical boundary conditions including simply supported beam, clamped beam, pinned-clamped beam and cantilever beam as well as shear hinge end condition are considered. The reduction of the frequencies in the presence of the lumped mass damper is observed. However, dimensionless frequency reduction is function of the lumped mass amount and lumped mass position, but the lumped mass position has more important effect on the vibration control of the functionally graded beam. The convergence of the numerical results is observed by using small terms of the basis.

2. Vibration analysis

The Figure (1) illustrates a functionally graded beam and an attached lumped mass damper. The length, width and height of the beam are L , b and h respectively. The position of the lumped mass damper is measured from the left end of the FGM beam. The mass of the ring will be considered in terms of the total mass of the FGM beam.

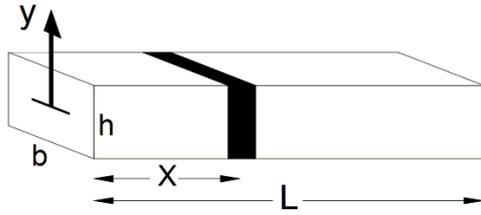


Figure 1. The FGM beam with lumped mass damper

The total potential energy of the functionally graded beam and connected lumped mass, m , at position X from left end of the beam is written as follows:

$$\Pi = \frac{1}{2} \int_0^L (EI_{eq}(w'')^2 - \bar{m}_{eq}w^2\omega^2)dx - \frac{1}{2}m(w|_{x=X}\omega)^2 \quad (4)$$

where, EI_{eq} and \bar{m}_{eq} are equivalent flexural rigidity and equivalent mass per unit length of the functionally graded beam. The deflection of the FGM beam is w . The parameter ω denotes natural angular frequency of the free vibration. In the Mori-Tanaka homogenization scheme the effective bulk modulus, K_e , and effective shear modulus, μ_e , can be calculated from Eq. (2) and Eq. (3).

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + \frac{V_m(K_c - K_m)}{K_m + \frac{4\mu_m}{3}}} \quad (2)$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + \frac{V_m(\mu_c - \mu_m)}{\mu_m + \frac{\mu_m(9K_m + 8\mu_m)}{6K_m + 12\mu_m}}} \quad (3)$$

The effective modulus of elasticity and Poisson's ratio are

$$E(y) = \frac{9K_e\mu_e}{3K_e + \mu_e} \quad (4)$$

$$\nu(y) = \frac{3K_e - 2\mu_e}{6K_e + 2\mu_e} \quad (5)$$

The distance between neutral axis position and mid-axis position is e . As a result, one can write

$$EI_{eq} = b \int_{-\frac{h}{2}-e}^{\frac{h}{2}-e} \left(\frac{9K_e\mu_e}{3K_e + \mu_e} \right) y^2 dy \quad (6)$$

The parameter e can be obtained as follows:

$$e = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} E(y) \times y dy}{\int_{-\frac{h}{2}}^{\frac{h}{2}} E(y) dy} \quad (7)$$

The mass density distribution of the FGM beam is assumed as follows:

$$\rho(y) = \rho_m V_m + \rho_c V_c \quad (8)$$

in which, ρ_m and ρ_c are mass density of the metal and ceramic constituents, respectively. The volume fraction of the phase materials are

$$V_c = \left(\frac{y}{h} + \frac{1}{2} \right)^n \quad (9)$$

$$V_m = 1 - \left(\frac{y}{h} + \frac{1}{2} \right)^n \quad (10)$$

where n is gradient index or material exponent parameter which takes non-negative real numbers. The linear mass density of the FGM beam is calculated as follows:

$$m_{eq} = b \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(y) dy \quad (11)$$

The spectral Ritz method is applied to calculate first frequencies of the system. The truncated Taylor series expansion of the deflection is selected as basis function and unknown coefficients.

$$w = \bar{w} + \sum_{k=4}^N c_k x^k \quad (12)$$

The function \bar{w} includes the coefficients c_0 to c_3 . After satisfying boundary and natural conditions, the coefficients c_0 to c_3 are calculated in terms of the remaining coefficients c_4 to c_N , consequently Eq.(12) is the general form of the deflection function for various end conditions of the FGM beam. For example, the function \bar{w} is calculated by satisfying $w(0) = w'(0) = w'(L) = W'''(L) = 0$ for clamped-shear hinge end FGM beam. Substituting Eq. (12) into Eq. (1), one has

$$\Pi = f(c_4, c_5, \dots, c_N) \quad (13)$$

Minimizing total potential energy, yields

$$\frac{\partial f}{\partial c_i} = 0 \quad 4 \leq i \leq N \quad (14)$$

The characteristic equation will be obtained by considering nontrivial solution. To this purpose, determinant of the matrix of the coefficients must be vanished.

3. Results and Discussion

The dimensionless fundamental angular frequency of the free vibration for simply supported FGM beam with various amounts of lumped mass against the position of the lumped mass is presented in Figure (2). The parameter M is the total mass of the FGM beam ($M = \bar{m}_{eq}L$). The dimensionless fundamental angular frequency of FGM beam for various amounts of lumped mass and various boundary conditions against the position of the lumped mass is presented in Figures (3) to (6).

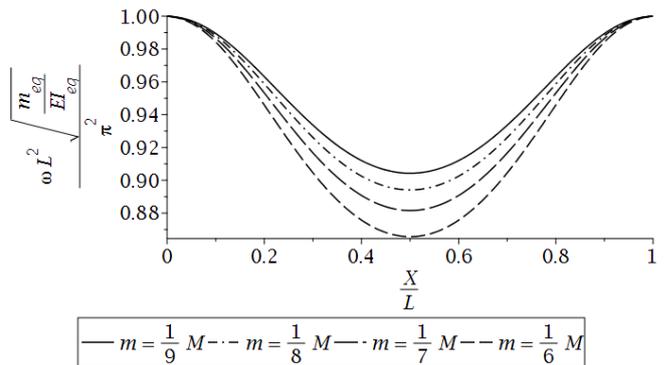


Figure 2. Dimensionless fundamental frequency of the simply supported FGM beam

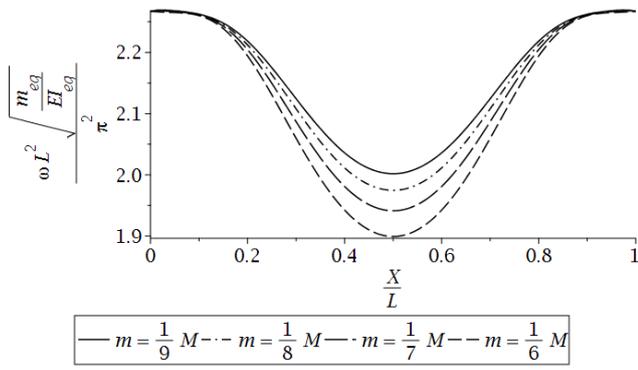


Figure 3. Dimensionless fundamental frequency of the clamped FGM beam

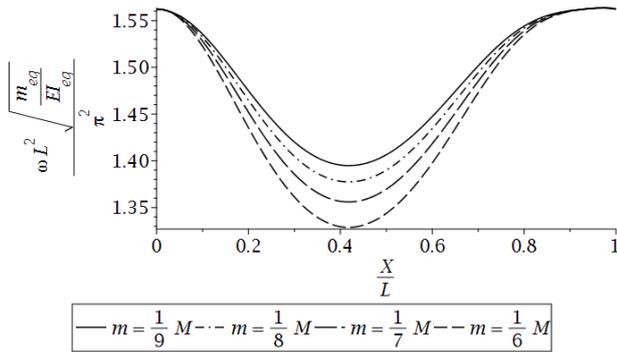


Figure 4. Dimensionless fundamental frequency of the pinned-clamped FGM beam

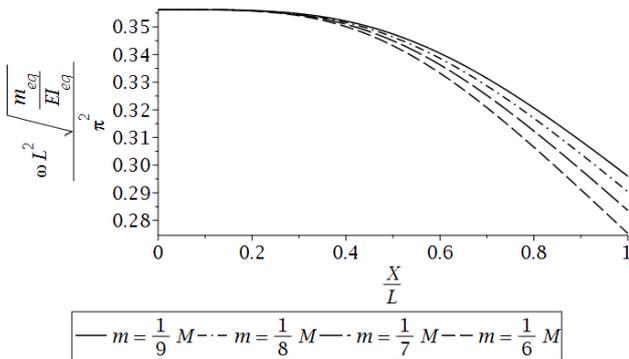


Figure 5. Dimensionless fundamental frequency of the cantilever FGM beam

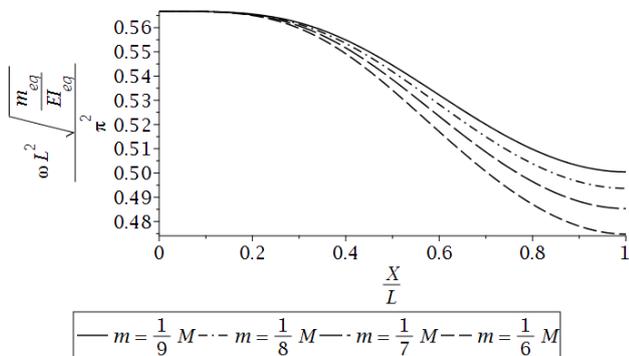


Figure 6. Dimensionless fundamental frequency of the clamped-shear hinge end FGM beam

Figures (2) to (6) show that the shape of the fundamental frequency reduction curve is similar to the FGM beam first mode shape. For example, first mode shapes of the pinned-clamped and clamped-shear hinge end FGM beams are presented in Figure (7).

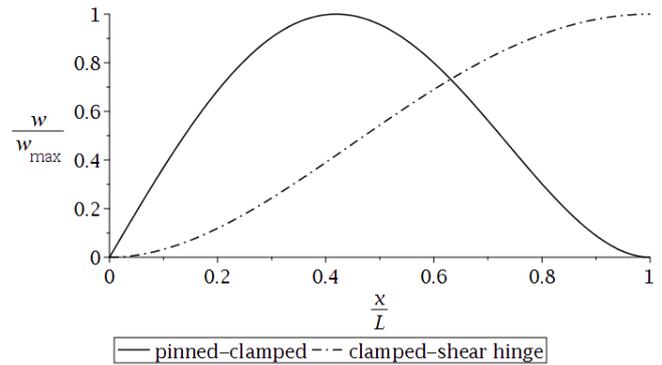


Figure 7. First mode shape of the FGM beam

4. Conclusion

The attached lumped mass position effect on the frequency reduction of the FGM beam is investigated. The equivalent linear mass density and equivalent flexural rigidity of the FGM beam are calculated. The results for various end conditions are presented schematically. Increasing connected lumped mass amount decreases dimensionless fundamental frequency. It is shown that the fundamental natural frequency reduction is function of the lumped mass amount and position. The frequency reduction diagram shape is similar to the mode shape of the FGM beam. The attached lumped mass can be used for vibration control of the FGM beam.

Nomenclature

- Π : The total potential energy
- EI_{eq} : The equivalent flexural rigidity
- \bar{m}_{eq} : The equivalent linear mass density
- M : The attached lumped mass
- X : The mass damper position
- w : The transverse deflection
- ω : The angular frequency of the vibration
- L : The length of the beam
- b : The width of the beam
- h : The height of the beam
- μ_e : The effective shear modulus
- K_e : The effective bulk modulus
- E : The elasticity modulus of the FGM beam
- ν : The Poisson's ratio
- e : The distance between neutral axis and mid-axis
- ρ_m : The mass density of the metal
- ρ_c : The mass density of the ceramic
- ρ : The mass density of the FGM beam
- V : The volume fraction of the phase material

Declaration of Conflict of Interests

The author declares that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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