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Research on numerical simulation of intelligent compaction of subgrade secondary development based on ABAQUS finite element

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Keywords

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Secondary development,
Controlled trials.

Abstract

It is an important method for theoretical research of intelligent compaction technology to establish and analyze the numerical simulation model of subgrade intelligent compaction. At present, the numerical simulation model of intelligent compaction for subgrade can not realize the dynamic change of subgrade parameters with the process of intelligent compaction. In order to realize the dynamic change of parameters in the process of intelligent compaction numerical simulation of subgrade, firstly, the relationships between the "field" - compaction degree and the internal friction Angle and cohesion of soil were obtained through the direct shear test in the laboratory. Secondly, the finite element of ABAQUS was redeveloped, and the dynamic change of subgrade parameters in the intelligent compaction process was realized by writing UMAT subroutine. Furthermore, the finite element model after redevelopment is compared with the model without redevelopment to verify the accuracy of finite element redevelopment. Finally, through the finite element secondary development of the numerical simulation model, a numerical simulation control test was designed to analyze the influence of different intelligent roller parameters on the final compaction degree. The conclusions obtained in this paper are as follows : (1) After the secondary development of ABAQUS finite element numerical simulation software, the final compaction degree is stable between 0.95 and 0.96, which can significantly improve the calculation accuracy of subgrade intelligent compaction process; (2) The influence of the input parameters of the intelligent compactor on the final compaction degree of the roadbed is ranked as follows: roller's weight > excitation frequency > excitation force amplitude.

1.Introduction

Compaction is an important part in road construction [1-3]. Road compaction conforming to the standards can significantly improve the strength, stiffness and stability of the road structure, so as to ensure the service life of road [4-6].Traditional compaction technology and quality detection test methods for compaction quality have effectively guaranteed the quality of road construction in a certain period of time. However, with the rapid development of revolutionary technology, its shortcomings are increasingly prominent [7,8]. As a result, intelligent compaction technology emerges as the times require. Intelligent compaction technology is a set of new compaction technology which integrates computer technology, high precision sensing technology and positioning technology. It can realize the comprehensive control of the intelligent roller, and transmit the current compaction state and feedback adjustment information to the users.

Although intelligent compaction technology has developed rapidly in recent years, especially in the analysis of intelligent compaction process through finite element numerical simulation, there are still the following problems : (1) The current research on the numerical simulation of intelligent compaction technology does not take into account the dynamic changes of mechanical parameters of the compacted structural layer [9,10].For example, in the process of intelligent compaction of subgrade, with the progress of compaction, various mechanical indexes of subgrade soil are changing, and its cohesion C and internal friction Angle φ will increase with the improvement of compaction quality. If it is not taken into account in the process of numerical simulation, the results of numerical simulation will be different from the actual situation, which will lead to errors in the results of numerical simulation.(2) At present, there are few studies on the influence of various factors on the final compaction degree in the process of intelligent compaction [11,12]. This will cause the mechanism of intelligent compaction technology to be blurred, and rely more on field test results to develop intelligent compaction technology, which leads to weak theoretical links of intelligent compaction.

In order to improve the accuracy of the numerical simulation of intelligent compaction, the influence degree of different influencing factors on the compaction quality was studied. In this paper, a simulation model of dynamic change of compaction parameters was established to study the influence trend and degree of different compaction parameters on compaction degree.

2. Research methods

Through the secondary development of the ABAQUS finite element, the intelligent compaction numerical simulation model was established to analyze the influence trend and degree of different compaction parameters (rolling speed, excitation force, weight) on the compaction degree, and to identify the key factors affecting the final compaction degree.

2.1. Finite element secondary development of ABAQUS[13,14]

In the process of finite element numerical simulation of subgrade compaction, the constitutive model and parameters of subgrade is the most basic and important part [15,16]. In this paper, the most common Moor-Coulomb elastoplastic model is chosen as the constitutive model in the process of subgrade compaction. In the Moor-Coulomb model, it is expressed by the shear strength index (cohesive force C, internal friction Angle φ) of soil. The established model needs to reflect the changes of the above-mentioned subgrade shear strength indexes. The specific implementation steps are as follows: firstly, the "field variable" in the numerical simulation is determined, which is the intermediate part to realize parameter correlation. In this paper, the compaction degree K is selected as the "field variable". Then, the relationship between the soil shear strength index and the "field variable", and the relationship between the soil strain and the "field variable " were established respectively. Finally, the relationship between soil strain and shear strength is realized by writing UMAT subroutine. Because the strain of soil can be obtained in real time in the finite element program, the real time variation of the shear strength index of soil can be realized. The relationship between the "field variable " and the subgrade strain and the relationship between the "field variable" and the shear strength index are described in detail in Section 2.1.1 and 2.1.2 respectively.

2.1.1 Establish the relationship between "field" and subgrade strain

In the process of compaction, the strain of subgrade can be correlated with the compaction degree [19], so the compaction degree can be calculated by soil strain. The following assumptions were made during the establishment of the simulation model:

(1) In the process of subgrade compaction, the horizontal and vertical strains are negligible compared with the model size, and the model shape in the process of compaction is considered as a rectangle.

(2) Soil is continuous, uniform and isotropic in the initial state and in the process of compaction.

In the model, the width of Z direction is set as 1, and no deformation occurs, that is, the strain ε3=0.The initial compaction is set to 0.8. The volume calculation process and expression of subgrade soil model in the compaction process are shown in (1).

$$\begin{aligned} V_1 &= x\,y\,z\left(1+\varepsilon_1\right)\ \left(1+\varepsilon_2\right)\left(1+\varepsilon_3\right) \\ &= x\,y\,z\left(1+\varepsilon_1\right)\ \left(1+\varepsilon_2\right)\left(1+0\right) \\ &= x\,y\,z\left(1+\varepsilon_1\right)\ \left(1+\varepsilon_2\right) \end{aligned} \quad (1)$$

Note: x -- the length of the model (m), the length of the model in this paper is 10m; y -- the thickness of the model's uncompacted layer (m). The thickness of the uncompacted layer in this paper is 0.3m; Z -- the width of the model (m), since the model is a two-dimensional model, z=1m; ε_1 -- strain in the x direction; ε_2 -- strain in the y direction; ε_3 -- the strain in the Z direction, ε_3=0.

Compaction degree K can be calculated by (2) :

$$K = \frac{\rho}{\rho_{max}} \quad (2)$$

Density ρ is calculated as (3) :

$$\rho = \frac{m}{v} \quad (3)$$

Note: K -- compaction degree; ρ -- the density corresponding to the current compaction degree K (kg/m^3); ρ_{max} -- the maximum dry density of the compacted material (kg/m^3); m -- mass corresponding to density ρ (kg); v -- volume corresponding to density ρ (m^3).

In the process of compaction, the mass m of soil is constant, the maximum dry density ρmax of the soil remains constant, and only the volume v and density ρ change in real time. Equation (4) can be obtained by simultaneous equations (1) (2) (3).

$$K = \frac{m}{x\,y\,z\left(1+\varepsilon_1\right)\ \left(1+\varepsilon_2\right)\rho_{max}} \quad (4)$$

At the same time, the initial density ρ0 can be calculated according to (5) :

$$\rho_0 = \frac{m}{x\,y\,z} \quad (5)$$

By combining (4) and (5), the expression of real-time compaction degree K can be obtained, as shown in (6) :

$$K = \frac{\rho_0}{\left(1+\varepsilon_1\right)\ \left(1+\varepsilon_2\right)\rho_{max}} \quad (6)$$

According to the initial setting of compaction degree K_1=0.8, the final relationship between real-time compaction degree K and strain can be obtained by substituting in (6), as shown in (7) :

$$K = \frac{0.8}{\left(1+\varepsilon_1\right)\ \left(1+\varepsilon_2\right)} \quad (7)$$

So far, the relationship between compaction degree K and the strain ε is determined by mathematical modeling.

2.1.2 Establish the relationship between the "field" and the soil shear strength index

In the previous study [17], author conducted direct shear tests on specimens with compaction degrees of 80%, 85%, 90%, 93%, 94%, 96% and 100%, and the test results are shown in Table 1:

Table 1. Test results of compaction degree, cohesion and internal friction Angle

Compaction degree	Cohesive c(kPa)	frictional angle φ(°)
0.8	12.1	27
0.85	41.2	28.8
0.9	58.6	31.2
0.93	61.6	30.5
0.94	62.8	32.4
0.96	63.6	34.3
1	74.9	36.9

Through fitting, the relations between compaction degree and cohesion c, and between compaction degree and internal friction Angle φ are as follows:

$$\begin{aligned} c &= -1248K^2 + 2532.8K - 1213.3 \\ \varphi &= 46.672K - 10.952 \end{aligned} \quad (8)$$

Thus, the relationship between strain and compaction degree, shear strength index and compaction degree is obtained, and the relationship between strain and shear strength index can be established in the finite element model. The initial compaction degree K=0.8, and the maximum compaction degree K_{max}=1. Through the fitting formula, 0.01 is taken as the step size to generate the one-to-one correspondence between the "field variable " compaction degree K and the soil shear strength, as shown in Table 2.

Table 2. The "field" compaction degree K corresponds to the shear strength of soil

Compaction degree	Cohesive c(kPa)	frictional angle φ(°)	Compaction degree	Cohesive c(kPa)	frictional angle φ(°)
0.8	26.4	14.2	0.91	31.5	58.1
0.81	26.9	19.5	0.92	32.0	60.6
0.82	27.3	24.4	0.93	32.5	62.8
0.83	27.8	29.2	0.94	32.9	64.8
0.84	28.3	33.7	0.95	33.4	66.5
0.85	28.7	37.9	0.96	33.9	68.0
0.86	29.2	41.9	0.97	34.3	69.3
0.87	29.7	45.6	0.98	34.8	70.3
0.88	30.1	49.1	0.99	35.3	71.0
0.89	30.6	52.4	1	35.7	71.5
0.9	31.1	55.3			

The data in Table 2 were added in the finite element numerical simulation of ABAQUS, as shown in Figure 1(a) and 1(b).

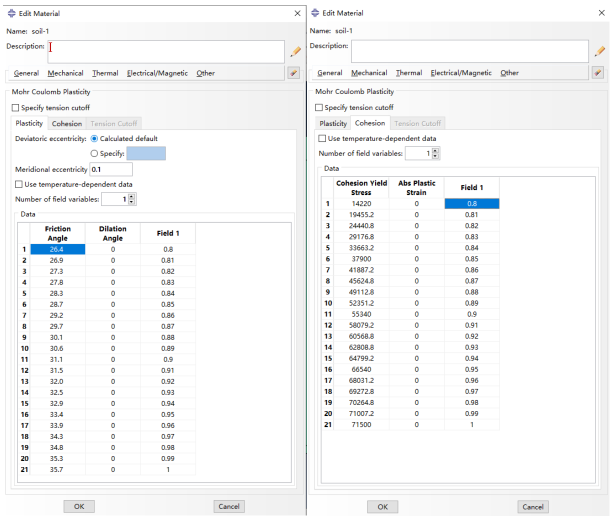

Fig.1 (a) Correlation method of "field" and cohesion C; (b) Correlation method of "field" and friction Angle φ

So far, it has been realized that in the numerical simulation, the shear strength of the roadbed changes in real time with compaction, thereby improving the accuracy of the numerical simulation.

2.2 Construction of subgrade compaction finite element model

2.2.1 Subgrade structure

The current research shows that the influence range of vibration wheel in the horizontal direction is about 3~5 m during the compaction process. Therefore, when establishing the numerical simulation model, the horizontal dimension is defined as 10m, which can eliminate the influence caused by the size effect. In the actual construction, the thickness of each loose layer of the roadbed is about 20 ~ 30 cm. Therefore, in this paper, the thickness of the upper loose pavement is defined as 30cm, and the lower foundation is defined as 3.7m. The subgrade structure diagram is shown in Figure 2. Table 3 shows the parameters of subgrade soil in the initial compaction state, and the construction parameters of intelligent compaction will be given in the design of test conditions below.

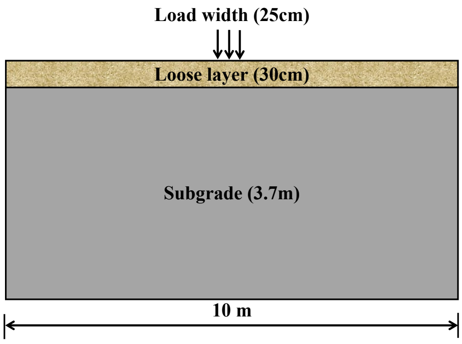

Figure 2. Roadbed structure

Table 3. Parameters of initial compaction state of subgrade soil

Structural layer	P (kg/m³)	E (MPa)	C (kPa)	Φ (°)	Poisson's ratio
Loose layer	1634	15	11.88	27	0.33
Subgrade	1859	30	55.8	31.15	0.35

2.2.2 Mesh size

The selection of mesh density is also a key part in numerical simulation. The selection of mesh density should not only ensure the accuracy of numerical simulation, but also ensure sufficient computational efficiency.

Relevant studies show that the mesh size is related to the calculation accuracy. In order to ensure the calculation accuracy, the mesh size should meet the following requirements:

$$\Delta l \le \left(\frac{1}{8}-\frac{1}{10}\right)\lambda \tag{9}$$

Note, λ -- the wavelength corresponding to the highest frequency.

The working range of excitation frequency of vibratory roller in site construction is mostly 25Hz~40Hz, and the corresponding highest frequency is about 40Hz. According to soil data in Table 3, the shear modulus of soil can be calculated according to (10).

$$G=\frac{E}{2(1+\mu)} \tag{10}$$

Note, G -- Shear modulus (MPa); E -- Elastic modulus (MPa); μ -- Poisson's ratio. The shear modulus of subgrade soil was calculated to be 5.64MPa.

The initial density of soil can be calculated according to (11) :

$$\rho=\rho_{\max}\cdot K \tag{11}$$

Note, ρ -- initial density (kg/m³);ρmax -- the maximum dry density (kg/m³);K is the initial compaction degree. The initial density of subgrade was calculated to be 1472 kg/m³.

In the process of compaction of subgrade, the vibration wave velocity can be calculated according to (12) :

$$c_s=\sqrt{\frac{G}{\rho}} \tag{12}$$

Note, cs -- vibration wave velocity (m/s).The vibration velocity of subgrade in the process of vibration compaction is 61.9m/s.

The wavelength corresponding to the highest frequency can be calculated according to (13) :

$$\lambda = \frac{c_s}{f} \tag{13}$$

Note, f -- the highest frequency (Hz). The length of vibration wave λ= 1.55m in the process of vibration compaction.

Thus, the mesh size of finite element numerical simulation is determined as follows:

△l≤0.16~0.19(m)

In this paper, calculation efficiency and accuracy are taken into account. The mesh size at the load center area is 0.05m, and the mesh size at the boundary is 0.2m. The fine mesh and the coarse mesh are connected by gradient mesh, as shown in Fig. 3.

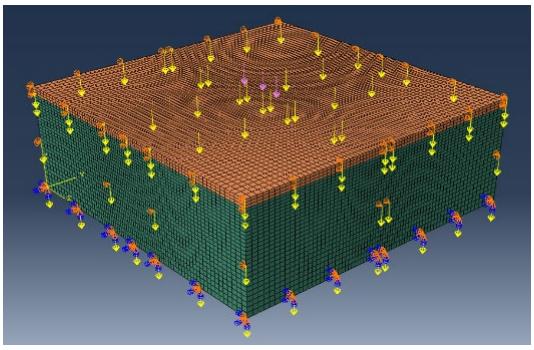

Figure 3. Finite element numerical simulation grid division diagram

2.3 Finite element numerical simulation working condition design

The weight of the roller, the amplitude of the excitation force, and the excitation frequency are based on the parameters of the Bomag BW 220 D-40 single-drum roller as the control group. On the basis of the weight of the roller M=20780kg, the excitation force amplitude P=314kN, and the excitation frequency f=32Hz, the experimental working condition table of three factors and five levels was established respectively. As shown in Table 4:

Table 4. Test table of three factors and five levels

Factors	Weight(kg)	Excitation force amplitude (kPa)	Excitation frequency (Hz)
NO.1	16624	251.2	25.6
NO.2	18702	286.2	28.8
NO.3	20780	314	32
NO.4	22858	345.4	35.2
NO.5	24936	376.8	38.4

Through the three factors and five variables designed in Table 4, the influence of single factor on the final compaction degree of subgrade is analyzed. Taking the NO.3 condition of each factor as the control group, 13 groups of test conditions can be obtained, as shown in Table 5:

Table 5. Single factor analysis of test conditions

Number	Weight (kg)	Excitation force amplitude (kPa)	Excitation frequency (Hz)
No.1	20780	314	32
No.2	16624	314	32
No.3	18702	314	32
No.4	22858	314	32
No.5	24936	314	32
No.6	20780	251.2	32
No.7	20780	286.2	32
No.8	20780	345.4	32
No.9	20780	376.8	32
No.10	20780	314	25.6
No.11	20780	314	28.8
No.12	20780	314	35.2
No.13	20780	314	38.4

According to the 13 groups of test conditions listed in Table 5, conditions 1-5 were selected to analyze the influence of the weight of roller on the compaction degree of subgrade. Working conditions 1 and 6-9 were selected to analyze the influence of excitation force amplitude on subgrade compaction degree. Working conditions 1 and 10~13 were selected to analyze the influence of vibration frequency on subgrade compaction degree.

In order to verify the accuracy of numerical simulation in the process of subgrade compaction in the secondary development of ABAQUS finite element numerical simulation, the development law of compaction degree over time was selected to compare the calculation results without subroutines. The numerical simulation without subroutine is completely consistent with the finite element model after secondary development in terms of model building, mesh division and load input. Only the difference exists in subgrade parameters. The subgrade parameters and vibratory roller parameters without subroutine directly refer to the parameters of working condition 1 in Table 3 and Table 5.

3. Results and analysis

3.1 Accuracy verification of numerical simulation

Since the road can be regarded as a semi-infinite space body, under the action of vibration compaction, the displacement change on its horizontal plane (XOY) is very small and can be ignored. Therefore, the change of vertical displacement U3 can be used to characterize the compactness.

In the numerical simulation, the initial compaction degree K of subgrade is 0.8, the thickness of loose layer is 0.3m, and the maximum compaction degree is 1. Therefore, real-time change of compactness with time can be calculated according to (14).

$$K = 0.8 + (1 - 0.8)\frac{U3}{(1-0.8)U} = 0.8 + 0.2\frac{U3}{U3max} \tag{14}$$

Note: U -- the thickness of loose layer, which is 0.3m in this paper; U_3 -- vertical displacement of subgrade (m); U_{3max} -- Theoretically the maximum vertical displacement can occur, 0.3m×(1-0.8)=0.06m.

Through the design of finite element numerical simulation control test, it is verified that the secondary development of ABAQUS finite element can effectively improve the accuracy of numerical simulation. The obtained test results are shown in Fig. 4.

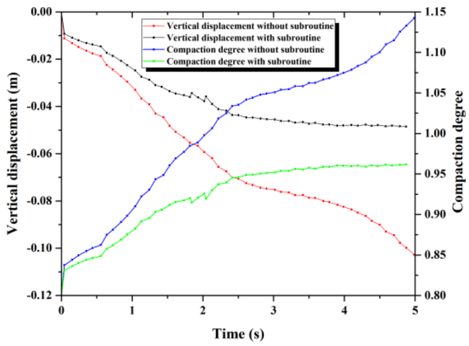

Figure 4. The development law of vertical displacement and compaction degree in subgrade compaction process with or without subroutine

As can be seen from the figure:

(i) It can be clearly seen from the law of compaction degree that the calculation result of the secondary development model of ABAQUS without subroutines is too large. The theoretical maximum value of the degree of compaction is 1, but the calculation result of the model without secondary development, the final degree of compaction is 1.14, which is obviously inconsistent with the actual situation. In contrast, the final compaction degree obtained through the secondary development of the model of finite element numerical simulation software is 0.965. After the subgrade compaction process is completed in strict accordance with the construction technology, the final compaction degree of the subgrade should reach more than 0.95, but it is usually not higher than 0.975, and 0.965∈[0.95,0.975]. The numerical simulation results are within a reasonable range, which verifies that the secondary development of finite element numerical simulation can improve the calculation accuracy.

(ii) From the curve of compaction degree without subroutine, except for the presence of a static compaction depth in the first stage, the compaction increases approximately linearly after that.. This is because in the model without subroutine, the default subgrade parameter is the initial parameter, which does not change with the growth of subgrade compaction quality. Therefore, under the premise of constant weight and excitation force of the roller, the compaction degree increases linearly. This is obviously not the case. Therefore, the second development of finite element numerical simulation can improve the accuracy of numerical simulation.

(iii) In addition, according to the test data after the secondary development of finite element numerical simulation, the law of compaction degree can be further divided into three stages, the initial stage of compaction, the process of compaction and the stage of compaction stability. In the first stage, the compaction degree increases rapidly from the initial 0.8 to about 0.83. The reason for this phenomenon is that when the roller comes into contact with the soil, the weight of the roller will increase the compaction degree of the subgrade to a certain extent, and the corresponding compaction depth is static compaction depth. In the second stage, the compaction degree of subgrade increases linearly. However, it is not difficult to find that the growth trend of compaction begins to slow down in the middle and late stage of the second stage, and finally, the compaction of subgrade exceeds 0.955 at the end of the second stage. The compaction degree of subgrade increases mainly in this stage. In the third stage, the compaction degree of subgrade tends to be stable and almost no longer increases. This is because the subgrade has reached the maximum compaction degree under the current construction technology, and the density of the subgrade is close to the maximum dry density. The final compaction degree of the subgrade is stable at 0.962.

3.2 Analysis of factors affecting the final compaction degree of subgrade

In order to more clearly show the influence trend of various factors on subgrade compaction degree, the influence trend of different factors on compaction degree is plotted according to Table 6.

According to the working conditions designed in Table 5, the ABAQUS finite element numerical simulation model was used to simulate 13 groups of working conditions, and the corresponding vertical displacements were extracted. The compaction degree was further calculated according to (14). The variation law of compaction degree in Section 3.1 indicates that the compaction enters the third stage after 3s, and the compaction degree is in a relatively stable state at this time. Therefore, the numerical simulation results between 3s and 5s were selected to analyze the influence of different factors on the compaction degree. The numerical simulation results are shown in Table 6.

Table 6. Numerical simulation results

Number	Compaction degree	Number	Compaction degree
NO.1	0.962	NO.8	0.966
NO.2	0.948	NO.9	0.967
NO.3	0.956	NO.10	0.951
NO.4	0.967	NO.11	0.957
NO.5	0.971	NO.12	0.962
NO.6	0.952	NO.13	0.968
NO.7	0.958		

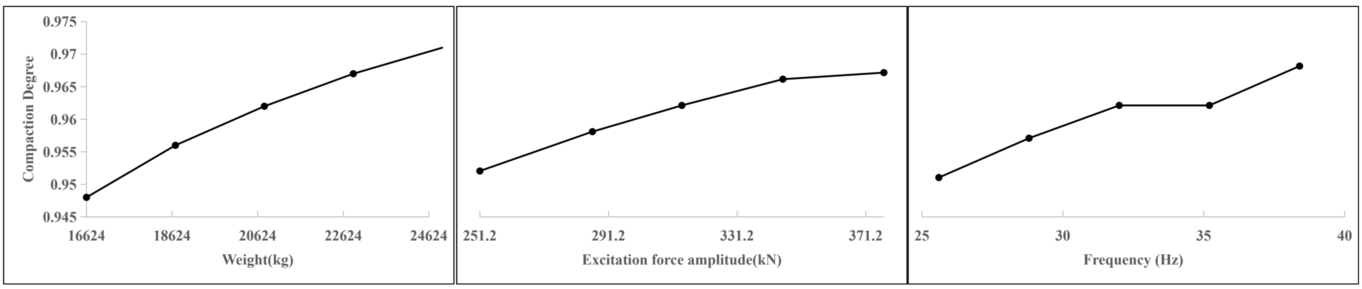

Figure 5. Influences of various factors on compaction trend

As can be seen from Figure 5:

(i) The impact of roller weight, excitation force amplitude and excitation frequency on the degree of compaction are all positive effects. That is, with the increase of the above three factors, the compaction degree also increases. The weight of the roller and the amplitude of the excitation force are positively correlated with the compaction quality of the subgrade, which is not difficult to understand. With the increase of the vertical load, the final compaction degree will inevitably increase correspondingly. In the finite element numerical simulation, with the increase of the excitation frequency, the compaction times of the roller also increase in the same time, and the total power by the roller on the subgrade increases, so the final compaction degree of the subgrade also increases.

(ii) Roller weight has the greatest influence on compaction degree. The maximum range of compaction degree produced by the weight of the roller is: $K=max(K_i)-min(K_i)=0.023$, which is also greater than the maximum range of compaction degree produced by the amplitude of the exciting force and the frequency of the exciting force is 0.015 and 0.017. From the range value, it can be seen that the range of weight is much larger than the range of exciting frequency and exciting force amplitude, which indicates that the influence of weight on compaction degree is much larger than the frequency and exciting force amplitude.

4、Conclusion

In this paper, through the secondary development of finite element numerical simulation, the accuracy of intelligent compaction numerical simulation of subgrade is effectively improved. Furthermore, a control test is designed to analyze the influence of various factors on the final compaction degree, and the following conclusions are drawn:

- After the secondary development of ABAQUS finite element numerical simulation, the final compaction degree is stable between 0.95 and 0.96, which can significantly improve the calculation accuracy of subgrade intelligent compaction process;
- The influence of the input parameters of the intelligent roller on the final compaction degree of the subgrade is weighted in the following order: the roller's weight > excitation frequency > excitation force amplitude.
- The three input parameters of the intelligent roller, the weight, the excitation frequency and the amplitude of the excitation force of the roller, all have positive effects on the final compaction degree.

Declaration of Conflict of Interests

The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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