A novel creep constitutive model based on Abel model for soil under cyclic loading

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Abstract

In geotechnical engineering, the creep problem of soil under cyclic loading is often involved. To describe the creep behavior of soil under cyclic loading, a novel creep constitutive model based on fractional Abel model, which describe the stable and destructive creep behaviors of soil under cyclic loading, was constructed. In this study, the cyclic loading was decomposed into a average static load and a cyclic loading with a zero average stress. The creep constitutive equation based on Abel model was given under the static load according to rheological mechanics theory, and the dynamic constitutive equation of soil under the decomposed loading was deduced according to viscoelastic mechanics theory. In the process, in order to reflect the influence of soil damage and plastic deformation on energy storage and energy consumption compliance of the ideal Abel model. The energy storage and consumption compliance variation parameters were introduced. Finally, a new creep constitutive equation of soil under cyclic loading could be obtained by superimposing the constitutive equation of soil under the decomposed static load and cyclic loading. The results showed the established creep constitutive model of soil under cyclic loading could be used to describe various creep behaviors of soil under cyclic loading, the model curves correlated very well with the experimental data, the obtained model parameters variation with increasing the dynamic stress amplitude of cyclic loading presented certain regularity, but the parameters change was discontinuous for stable creep and destructive behaviors.

Keywords

Soil mechanics, Cyclic loading, Abel model, Rheological mechanics, Viscoelastic mechanics, Creep behavior.

1. Introduction

In geotechnical engineering, the soil is often subjected to cyclic load, such as traffic load [1], machine vibrations [2] and the others. However, under long-term cyclic loading, the strength of soil would decrease and the deformation increased, thus some engineering accidents maybe be caused. The creep behavior of soil under cyclic loading are often encountered especially in subgrade and tunnel engineering, for instance, the tunnel settlement of subways in Shanghai has exceeded 20 cm since its operation [3], and the subsequent settlement caused by traffic load has reached 15cm according to the field survey of Saga airport [3, 4]. Therefore, the creep behavior of soil under cyclic loading are related to the safety of engineering construction.

The strength and creep properties of soils under cyclic loading are not only related to the properties of soil, but also to the frequency, stress amplitude and loading time of cyclic loading. When the stress upper limit of cyclic load is higher than the critical strength of soil, the destructive creep occurs; When the stress upper limit of cyclic load is lower than the critical strength, the stable creep occurs; When the stress upper limit of cyclic loading is near the critical strength, the critical creep occurs (see Fig. 1) [5]. To predict and calculate the permanent deformation of soils under cyclic loading, the some creep constitutive model for soils under cyclic loading are proposed, the two most common creep constitutive models are empirical constitutive and component model.

There are empirical models for predicting and calculating the permanent deformation of soils under cyclic loads. Power model \( \varepsilon_p = \alpha N^\beta \) [6], modified or empirical power models [7-10], such as a modified exponential model \( \varepsilon_p = \varepsilon_0 \cdot e^{\left(1000\sigma/\sigma_{crr}\right)^b} \) proposed by Guo [11]. Also, there are some theory model such as anisotropic hardening model[12,13], boundary model and the others. Additionally, Ren [14] established a new strain accumulation model \( \varepsilon_p = \frac{\sigma_{crr}}{\sigma_{crr} \cdot 1000} \) with the three parameters by analogizing the Hardin-Drnevich model and Monismith model. Jia [15] proposed a cumulative strain prediction model for soils under high and low cyclic stress based on classical elastic-plastic theory. Ren [16] proposed a grey cumulative plastic deformation model NNGM (1,1) by improving the traditional grey model GM (1,1) to predict the permanent deformation of soil under cyclic loading.
In terms of component models, according to the rheological theory, many models, which can describe different creep behavior of soils, can be established by combining elastic, viscous and plastic elements. Huang et al. [17] established a creep constitutive model to describe the creep behavior of soil under cyclic loading based on Kelvin model by simplifying the random loading on the pavement as a harmonic load. Additionally, many constitutive models are built by simplifying the cyclic load to a static load. In fact, the creep curve of behavior of soil. Furthermore, many constitutive models are built by simplifying the cyclic load to a static load, the cyclic loading is transformed into a static stress by the principle of impulse equivalence in the process of modeling. Chen [19] used the elastic, viscous and plastic fatigue elements to establish a fractional-order equivalence in the process of modeling. The sinusoidal cyclic loading was taken as a research object, and it was assumed that the soil is subjected to the cyclic loading shown in Fig.3, where

\[ \sigma = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}, \sigma_d(t) = \sigma_{\text{amp}} \sin \omega t \]  (2)

\[ \varepsilon(N) = \varepsilon_d(N) + \varepsilon_s(N) \]  (3)

Where \( \varepsilon(N) \) is strain of soil under cyclic loading \( \sigma(t) \) when the cycle number of cyclic loading is \( N \); \( \varepsilon_d(N) \) is dynamic strain of soil under cyclic loading \( \sigma_d(t) \); \( \varepsilon_s(N) \) is static strain of soil under static load \( \sigma \) when the cycles number of cyclic loading \( \sigma_d(t) \) was \( N \).

2. Creep theory of soil under cyclic loading

2.1. Cyclic loading

The sinusoidal cyclic loading was taken as a research object, and it was assumed that the soil is subjected to the cyclic loading shown in Fig.3, where the cyclic loading can be expressed as

\[ \sigma(t) = \frac{[\sigma_{\text{max}} + \sigma_{\text{min}}]/2 + ([\sigma_{\text{max}} - \sigma_{\text{min}}]/2) \sin \omega t} \]  (1)

Where \( \sigma_{\text{max}} \) is the upper limit of cyclic loading, \( \sigma_{\text{min}} \) is the lower limit of cyclic loading, \( f \) is the frequency of cyclic loading, \( f_1 = 1/T \) is the time of cyclic loading acting on soil, the relationship between the number of cycles \( N \) and time \( t \) is \( t = NT \), \( \omega \) is angle frequency. \( \omega = 2\pi f \). Stress amplitude \( \sigma_{\text{amp}} \) of cyclic loading is \( (\sigma_{\text{max}} - \sigma_{\text{min}})/2 \).

2.2 Creep behavior of soil under cyclic loading

The creep behavior of soil under cyclic loading can be divided into stable, critical and destructive creep according to the curves \( \varepsilon(N) - N \). According to previous study [5, 20], in order to study the creep behavior of soil under cyclic loading, the cyclic loading \( \sigma(t) \) was decomposed into a static load \( \sigma \) and cyclic loading \( \sigma_d(t) \) with an average of 0 (see Fig.4). The creep behavior of soil under cyclic loading can be divided into stable, critical and destructive creep according to the curves \( \varepsilon(N) - N \).

According to previous study [5, 20], in order to study the creep behavior of soil under cyclic loading, the cyclic loading \( \sigma(t) \) was decomposed into a static load \( \sigma \) and cyclic loading \( \sigma_d(t) \) with an average of 0 (see Fig.4).
3. Abel model under cyclic loading

Based on the rheological mechanics theory, the mechanical model of elastic material is represented by spring, and its constitutive equation under cyclic loading is $\sigma(N) = E\varepsilon(N)$. The mechanical model of viscous material is represented by the dashpot with piston, and its constitutive equation under cyclic loading is $\sigma = \eta|d\varepsilon(N)/dN|$. The constitutive equation of elastic material can be written as $\sigma = E[d^2\varepsilon(N)/dN^2]$, the constitutive equation of viscous material can be written as $\sigma = \eta[d^2\varepsilon(N)/dN^2]$. However, there is a model which can be described as the mechanical property of viscoelastic material, it is the fractional-order Abel model shown in Fig. 6. The Abel model is the fractional-order creep constitutive equation based on Abel model (see Fig.4).

$$\sigma(t) = \eta \frac{\sigma(t)}{\eta} + \frac{n}{m} \frac{d^2\varepsilon(t)}{dt^2} (4)$$

Where $\eta$ is the viscoelastic coefficient of Abel model, whose physical dimension is [stress - number]. $n$ is the material parameter reflecting the creep rate of soil under load, the parameter $\eta, n$ could be determined by test.

4. Creep constitutive model of soil under cyclic loading $\sigma(t)$

4.1 Creep constitutive model under static load $\sigma$

When $\sigma(t) = \sigma$, according to the basic fractional calculus theory, the creep constitutive equation Eq. (5) of Abel model shown in Fig. 6 can be obtained from Eq. (4).

$$\varepsilon(N) = \frac{\sigma}{\eta}\frac{n}{t^m} (5)$$

The Eq.(5) is the fractional-order creep constitutive equation based on Abel model. In order to discuss the property of Eq.(5), it is supposed that $\sigma = 12$ MPa, $\eta = 45Pa\times times$, $N$ ranged from 1 to 1000. When $n$ was taken as 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 respectively, and the viscous coefficient $\eta$ was constant value, the influence of the the parameter $n$ variation on the creep curve $\varepsilon(N) - N$ could be obtained by substituting $n$ into Eq. (5). Fig.7 shows that Eq.(5) could be used to describe stable and destructive creep of soil when different parameter $n$ values were chosen.

4.2 $\sigma_d(t)$ under cyclic loading $\sigma_d(t)$

According to viscoelastic mechanics [28, 29], the creep process of soil under cyclic loading is a constant process of energy storage and consumption. The strain response of elastic solid material changes with stress in the same phase. For ideal viscous material, the phase difference between strain and stress is $\pi/2\omega$ (angular frequency) . However, the rock and soil is not ideal elastic material or viscous material, but viscoelastic-plastic material, so the soil strain under cyclic loading and the stress it is subjected to is not in the same phase or $\pi/2\omega$, the strain phase lags behind the stress it is subjected to. For viscoelastic-plastic material such as rock and soil, the phase angle $\phi$ between stress and strain is between 0 and $\pi/2\omega$. In this article, it is assumed that the phase difference between stress and strain of soil is $\phi$, which is also called the phase angle or energy dissipation angle between strain and stress it is subjected to.

If is assumed that at time $t$, the complex form of dynamic load $\sigma_d(t)$ in complex plane is

$$\sigma_d(t) = \sigma = \sigma_{ampl}(\omega t + \omega(t)) = \sigma_{ampl}(\omega t + \omega(t)) + isin(\omega t + \omega(t)) = \sigma_{ampl}(\omega t + \omega(t)) + isin(\omega t + \omega(t))$$

Where $\sigma_{ampl}$ is the stress amplitude of cyclic loading, $i$ is an imaginary unit.

According to the theory of viscoelastic mechanics, the dynamic strain $\varepsilon_d(t)$ of soil under oscillating stress $\sigma_d(t)$ is

$$\varepsilon_d(t) = \varepsilon_{ampl}[\cos(\omega t + \phi) + \sin(\omega t + \phi)] = \varepsilon_{ampl}[\cos(\omega t + \phi) + \sin(\omega t + \phi)]$$

Where $\varepsilon_{ampl}$ is the amplitude of strain, which can be solved from the $\varepsilon_{ampl}$ is the response complex strain amplitude, where $\phi$ is the energy dissipation angle, which can be obtained from Eq. (9)[20].

$$\phi = \arctan(J_1/J_2) (9)$$

Where $J_1$ is energy storage compliance of model, $J_2$ is energy consumption compliance.

For fractional-order Abel model, under cyclic loading $\sigma_d(t)$, the constitutive based Abel model could be obtained from Eq.(4).

$$\sigma_{ampl} = \frac{\varepsilon_{ampl}}{\eta\varepsilon^{(ampl}(\omega t) + i\sin(\omega t + \phi) = \varepsilon^{ampl}(\omega t) + i\sin(\omega t + \phi) = \varepsilon^{ampl}(\omega t) + i\sin(\omega t + \phi)$$

From Eq(10), we have

$$\frac{\varepsilon_{ampl}}{\eta}\varepsilon^{(ampl}(\omega t) = J_1 - J_2 (11)$$

With $I^m = \cos(\pi/2) + \sin(\pi/2)$, the Eq. (11) can be written as

$$\frac{\varepsilon_{ampl}}{\eta}\varepsilon^{(ampl}(\omega t) = J_1 - J_2 (13)$$

From Eq.(11), the energy storage compliance $J_1$ of Abel model is $[\varepsilon^{(ampl}(\pi/2)]/\eta\omega^n$ and the energy consumption compliance $J_2$ is $[\varepsilon^{(ampl}(\pi/2)]/\eta\omega^n$.
Soil is generally heterogeneous and anisotropic material containing many voids. Under dynamic loading, the plastic deformation and damage appeared in the soil, resulting in that energy storage compliance and energy consumption compliance of the material are different from the ideal model. In this paper, as Pu’s study [20,26], it is assumed that the energy storage compliance and energy consumption compliance caused by soil damage in the process of loading and unloading are $\Delta J_1$ and $\Delta J_2$, respectively. $\Delta J_1$ and $\Delta J_2$ values can be determined by experiments, which can reflect the changes of energy storage compliance and energy consumption compliance of soil in the process of loading and unloading. Consequently, the energy storage compliance and energy consumption compliance based on Abel mode could be written as Eqs. (14) and (15) respectively.

$$J_{A1} = \frac{\cos(\pi N/k)+a}{\cos(\pi a/k)} + \Delta J_{A1} \tag{14}$$

$$J_{A2} = \frac{\sin(\pi N/k)+a}{\sin(\pi a/k)} + \Delta J_{A2} \tag{15}$$

Where $\Delta J_{A1}$ is the change of energy storage compliance of Abel model caused by soil damage, its physical dimension is [stress $^{-1}$], $\Delta J_{A2}$ is the variation of energy dissipation compliance of Abel model, and its physical dimension is the same as that of $\Delta J_{A1}$.

From Eq.(7), the dynamic strain $\varepsilon_d(t)$ of fractional-order Abel model under cyclic loading $\sigma_d(t)$ in the form of sine and cosine wave are as follows.

$$\varepsilon_d(t) = \varepsilon_d(N) = \sigma_{ampl}\sqrt{J_{A1}+J_{A2}} \sin \left(\frac{\pi N}{k} + \arctan \left(\frac{\pi a}{k}\right)\right) -\arctan \left(\frac{\pi a}{k}\right) - \varepsilon_0 \tag{16}$$

$$\varepsilon_d(t) = \varepsilon_d(N) = \sigma_{ampl}\sqrt{J_{A1}+J_{A2}} \cos \left(\frac{\pi N}{k} + \arctan \left(\frac{\pi a}{k}\right)\right) - \arctan \left(\frac{\pi a}{k}\right) - \varepsilon_0 \tag{17}$$

The Eqs.(16) and (17) could be used to describe the strain fluctuation range of soil under cyclic loading, and the fluctuation range is

$$\left(-\sigma_{ampl}\sqrt{J_{A1}+J_{A2}}, \sigma_{ampl}\sqrt{J_{A1}+J_{A2}}\right) \tag{18}$$

### 4.3 Creep constitutive equation of soil under cyclic loading

The stable and critical creep behavior of soil are described by the proposed model based on Abel model, and the constitutive equation Eq.(19) and (20) for soil under cyclic loading $\sigma(t)$ in the form of sine and cosine wave could be obtained by superimposing Eqs.(5), (16), and (17).

$$\varepsilon(N) = \sigma_{ampl}\sqrt{J_{A1}+J_{A2}} \sin \left(\frac{\pi N}{k} + \arctan \left(\frac{\pi a}{k}\right)\right) + \sigma_N \eta \frac{1}{1+n} \tag{19}$$

$$\varepsilon(N) = \sigma_{ampl}\sqrt{J_{A1}+J_{A2}} \cos \left(\frac{\pi N}{k} + \arctan \left(\frac{\pi a}{k}\right)\right) + \sigma_N \eta \frac{1}{1+n} \tag{20}$$

The Eqs.(19), (20) are centered on the non-linear function $P(\eta , n)$ with a fluctuation range of $\left(-\sigma_{ampl}\sqrt{J_{A1}+J_{A2}}, \sigma_{ampl}\sqrt{J_{A1}+J_{A2}}\right)$. The Eqs.(19) and (20) can reflect the stable and critical creep behavior of soil under sine and cosine cyclic loading when different parameter $n$ values are chosen.

### 5. Parameter and adaptability verification of model

In this paper, based on the existing experimental data, the differential evolution (DE) method in 1stOpt software [20] is used to calculate the parameters of the proposed constitutive Eq.(19) and to fit the curve to verify the adaptability of the proposed model.

### 5.1 Model curves and parameters of red-mudstone soil

Kong et al. [21] to carry out cyclic tests on red mudstone soil under sinusoidal cyclic loading with frequency $f = 5Hz$ by using the GDS dynamic triaxial testing system. From this test, the critical dynamic stress amplitude $\sigma_{dc}$ of red-mudstone soil under 25 kPa confining pressure was 216 kPa, and the static shear strength $\tau_f$ was 720 kPa. In this test, the vibration center of axial dynamic stress is $\sigma = \tau_f + \sigma_{dc}$, and the axial deviatoric stress can be expressed as Eq.(21). In this paper, the creep curves of red-mudstone soil under 25kPa confining pressure were fitted and model parameters were obtained by using Eq.(19). The obtained parameter results are shown in Table 1 and the creep curves of the proposed model are shown in Fig.8.

$$\sigma(t) = 2\sigma_{ampl} + \sigma_{ampl}\sin(\omega t) \tag{21}$$

Where $\sigma_{ampl}$ is the stress amplitude of cyclic loading.

It can be seen from Table 1 that the parameters of the proposed model varied in a small range and exhibited an obvious regularity. The viscosity coefficient $\eta$ of the proposed model decreased with increasing dynamic stress amplitude $\sigma_{dc}$ of cyclic loading (see Fig. 9). Meanwhile, the $\Delta J_{A1}$ and $\Delta J_{A2}$ of energy storage compliance and energy consumption compliance were less than 0, indicating that there was a decrease in the energy storage and energy consumption compliance of soil under dynamic loading due to soil damage. When $\sigma_{dc} < \sigma_{dc}$, the $\Delta J_{A1}$ evolution of energy storage compliance of soil with dynamic stress amplitude $\sigma_{dc}$ is shown in Fig. 10, and the variation $\Delta J_{A2}$ of energy consumption compliance decreased with increasing dynamic stress amplitude $\sigma_{dc}$ of cyclic loading (see Fig 13), but the change rule of parameter $n$ is not obvious.

According to the model curves of red-mudstone soil and roadbed coarse-grained soil, the creep constitutive equation established based on Abel model could be used to describe the creep behavior of soil under cyclic loading, including stable and destructive creep behavior. The model curve was correlated with well creep curves from experimental data, the correlation coefficients were all above 0.900.

### Table 1. Model parameters from red-mudstone soils under 25 kPa confining pressure [21].

<table>
<thead>
<tr>
<th>$\sigma_d$ (kPa)</th>
<th>$\sigma_{dc}$ (kPa)</th>
<th>$\eta$ (kPa-number of times)</th>
<th>$\Delta J_{A1}$ (kPa)</th>
<th>$\Delta J_{A2}$ (kPa)</th>
<th>n</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>216</td>
<td>1.180×10^4</td>
<td>-5.128×10^4</td>
<td>-1.136×10^5</td>
<td>0.139</td>
<td>0.9375</td>
</tr>
<tr>
<td>45</td>
<td></td>
<td>6.789×10^1</td>
<td>-1.098×10^4</td>
<td>-1.875×10^3</td>
<td>0.126</td>
<td>0.9824</td>
</tr>
<tr>
<td>65</td>
<td></td>
<td>4.818×10^1</td>
<td>-1.487×10^4</td>
<td>-2.558×10^5</td>
<td>0.119</td>
<td>0.9784</td>
</tr>
<tr>
<td>95</td>
<td></td>
<td>3.528×10^1</td>
<td>-1.899×10^4</td>
<td>-3.391×10^5</td>
<td>0.113</td>
<td>0.958</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>2.360×10^1</td>
<td>-3.154×10^4</td>
<td>-4.427×10^5</td>
<td>0.091</td>
<td>0.958</td>
</tr>
<tr>
<td>225</td>
<td></td>
<td>1.375×10^2</td>
<td>-4.865×10^4</td>
<td>-8.649×10^5</td>
<td>0.112</td>
<td>0.958</td>
</tr>
<tr>
<td>245</td>
<td></td>
<td>2.205×10^2</td>
<td>-1.265×10^4</td>
<td>-7.204×10^5</td>
<td>0.330</td>
<td>0.999</td>
</tr>
<tr>
<td>260</td>
<td></td>
<td>1.784×10^3</td>
<td>-1.264×10^4</td>
<td>-7.724×10^5</td>
<td>0.449</td>
<td>0.999</td>
</tr>
</tbody>
</table>
5.2 Model curve and parameters of saturated coarse-grained soil

Zhou [30] et al conducted on a series of dynamic triaxial tests on coarse-grained soil with consolidation ratio of 1.0 under the cyclic loading shown in Fig.12. In this test, the dynamic stress amplitude \( \sigma_d \) (i.e. \( \sigma_{amp} \)) of the cyclic loading is 25 ~ 125 kPa, and the sinusoidal cyclic loading frequency is 1 Hz. In Fig.12, the OA is the consolidation pressure (i.e. confining pressure) and \( \sigma_s \) is the static deviatoric stress, \( \sigma_s \) is taken as 15 kPa in the test. Then the deviatoric stress acting on the specimen is shown in Eq.(22). In this paper, the established constitutive Eq.(19) was used to fit creep curve and obtain the model parameters of coarse-grained soil under 15 kPa confining pressure.

The obtained parameters and model curves are shown in Table 2 and Fig. 13, respectively.

<table>
<thead>
<tr>
<th>( \sigma_d ) (kPa)</th>
<th>( \sigma_{dc} ) (kPa)</th>
<th>( n )</th>
<th>( \Delta J_{A1} ) (kPa-1)</th>
<th>( \Delta J_{A2} ) (kPa-1)</th>
<th>( \eta ) (kPa·number of times)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>50</td>
<td>0.070</td>
<td>0.904</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>4.421×10^2</td>
<td>0.332</td>
<td>0.995</td>
<td></td>
<td></td>
</tr>
<tr>
<td>62.5</td>
<td>7.705×10^2</td>
<td>0.453</td>
<td>0.999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>7.640×10^2</td>
<td>0.420</td>
<td>0.999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>5.389×10^2</td>
<td>0.478</td>
<td>0.996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>3.539×10^2</td>
<td>0.478</td>
<td>0.996</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \frac{\partial \sigma}{\partial n} = -0.189(\sigma_s - 24.992)^{0.204} \)
\( \text{R-square} = 0.955 \)
and each parameter of the soil decreased with the increasing dynamic stress, the parameters of the model had obvious change rules, from this study.

In order to study the creep characteristics of soils under cyclic loading, a new creep constitutive model of soils under cyclic loading was established, and the proposed model could be used to describe the different creep behavior of soils under cyclic loading.

6. Conclusions

In order to study the creep characteristics of soils under cyclic loading, a new creep constitutive model of soils under cyclic loading was derived based on Abel model, the following conclusions are obtained from this study.

(1) Based on Abel model, a creep constitutive model which can describe the stable, critical and destructive creep behavior of soil under cyclic loading, was established, and the proposed model had only three parameters.

(2) When the different values of parameter \( n \) are chosen, the model could be used to describe the different creep behavior of soils under cyclic loading.

(3) The established creep constitutive model based on Abel model was used to fit the experimental data from different soils, the results show that the proposed model constitutive equation was correlated well with experimental data.

(4) The obtained parameter \( n \) of model increased with increasing dynamic stress amplitude \( \sigma_d \) of cyclic loading, while the other parameters decreased with the dynamic stress amplitude \( \sigma_d \) increasing. However, the attenuation law of parameters under high and low stress was consistent.

Declaration of Conflicts of Interest

The authors declare that there is no conflict of interest.

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